VSM COLLEGE
(AUTONOMOUS)

RAMACHANDRAPURAM, E.G.Dt. (A.P.)

Re-Accredited by NAAC with 'B' Grade

V.S.M. COLLEGE

RAMACHANDRAPURAM

TEACHIG NOTES
2019-2020

Name of the Department / Subject : MATHEMATICS

Name of the Lecturer : N.S.V. KIRAN KUMAR

Course/Group:	III BSC
Paper:	V
Name of the Topic	Subrings, Ideals, Quotient rings.
Hours required	11.
Learning Objectives	properties of groups, Ideals, Quotient ring
Previous knowledge to be reminded	Properties of groups.
Topic Synopsis	Definition of subrings, Ideals, principal Ideal. (Continue on the reverse side if needed) principal Ideal ring. Quotient rings (or) Factor rings.
Examples/Illustrations	Given on the Reverise side.
Additional inputs	
Teaching Aids used	Black board.
References cited	Telugu academs and Schand.
Student Activity planned after the teaching	Assignments and Seminars.
Activity planned outside the Class room, if any	
Any other activity	

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* **Sub ring** :- Let $(R,+,\cdot)$ be a ring and S be a non-empty sub set of R. If $(S,+,\cdot)$ is also a ring w.r. to the two operations $+,\cdot$ in R. Then $(S,+,\cdot)$ is a subring of R.

* **Definition** :- Let $(F,+,\cdot)$ be a field and $(S,+,\cdot)$ be a subring of F. If $(S,+,\cdot)$ is a field. Then we say that S is a sub field of F. If $(S,+,\cdot)$ is an I.D. then we say that S is a sub domain of F.

* **Ideals** :- Let $(R,+,\cdot)$ be a ring. A non-empty subset U of R is called a two sided ideal or ideal if.

i) $a,b \in U \Rightarrow a-b \in U$, and

ii) $a \in U$ & $r \in R \Rightarrow ar, ra \in U$.

* **Definition** :- A non-empty subset U of a Ring R is called a right ideal if i) $a,b \in U \Rightarrow a-b \in U$ &

ii) $a \in U, r \in R \Rightarrow ar \in U$

A non-empty subset U of a ring R is called a right ideal if i) $a,b \in U \Rightarrow a-b \in U$ &

ii) $a \in U, r \in R \Rightarrow ar \in U$.

A non empty subset U of a ring R is called a ~~right~~ left ideal if i) $a,b \in U \Rightarrow a-b \in U$ &

ii) $a \in U, r \in R \Rightarrow ra \in U$.

* **Principal Ideal :-**

Let R be a commutative ring with unity and $a \in R$. The ideal $\{ra / r \in R\}$ of all multiples of 'a' is called the principal Ideal generates by 'a' and is denoted by (a) or $\langle a \rangle$

* **principal Ideal ring :-**

A commutative Ring R with unity is a principal Ideal ring if every ideal in R is a principal Ideal.

* **Quotient Rings (or) Factor rings :-**

Let R be a ring and U be an Ideal of R. Then the set $R/U = \{x + U / x \in R\}$ w.r. to induced operations of addition and multiplication of cosets defined by $(a+U) + (b+U) = (a+b) + U$, $(a+U)(b+U) = ab + U$ for $a+U, b+U \in R/U$ is a Ring, this ring $(R/U, +, \cdot)$ is called the Quotient ring (or) Factor ring (or) Residue class ring of R modulo U.

Question Bank

1) Let S be a non-empty subset of a Ring R. Then S is a subring of R. if $a-b \in S$ & $a, b \in S$.

2) The intersection of two Subrings of a Ring R is a subring of R.

3) A field has no proper non-trivial ideals (or) the ideals of a field F are only $\{0\}$ and F itself.

4) If R is a commutative Ring and $a \in R$. then $Ra \{ra / r \in R\}$ is an ideal of R.

5) A commutative ring R with unity element to a field, if R have no proper ideals.

6) The intersection of two ideals of Ring R is an ideal of R.

7) If U_1 and U_2 are two ideals of a ring R then $U_1 \cup U_2$ is an ideal of R if $U_1 \subset U_2$ (or) $U_2 \subset U_1$

8) If U_1, U_2 are two ideals of a ring R. Then $U_1 + U_2 = \{x+y / x \in U_1, y \in U_2\}$ is also an ideal of $x \in R$.

9) The ring of integers Z is a principal Ideal ring (or)

Every Ideal of Z is a principal Ideal.

Course/Group:	BCA
Paper:	I
Name of the Topic	Interpolation and finite differences
Hours required	10
Learning Objectives	Interiplatione, forward differences & backward defferences
Previous knowledge to be reminded	
Topic Synopsis	Difference tables forward differences Backward differences Newton's forward interpolation formula (Continue on the reverse side if needed) Newton's backward interpolation formula
Examples/Illustrations	Given on the reverse side
Additional inputs	
Teaching Aids used	Black board
References cited	Telugu acadamy, S. chand
Student Activity planned after the teaching	Assignments and seminars
Activity planned outside the Class room, if any	
Any other activity	

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Forward differences

Consider a function $y = f(x)$ of an independent variable x. Let $y_0, y_1, y_2 --- y_r$ be the value of y corresponding to the values $x_0, x_1, x_2 --- x_r$ of x respectively. Then the difference $y_1 - y_0$, $y_2 - y_1$ --- are called the first forward difference of y, and we denote then by $\Delta y_0, \Delta y_1$ ----

that is $\Delta y_0 = y_1 - y_0, \Delta y_1 = y_2 - y_1, \Delta y_2 = y_3 - y_2$ ----

Ingeneral $\Delta y_r = y_{r+1} - y_r \therefore r = 0, 1, 2, ---$ Here the symbol Δ is called forward difference operator.

The differences of the first forward differences are called second forward differences and are denoted by $\Delta^2 y_0, \Delta^2 y_1$, --- that is

$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$, $\Delta^2 y_1 = \Delta y_2 - \Delta y_1$. Ingeneral $\Delta^2 y_r = \Delta y_{r+1} - \Delta y_r$, $r = 0, 1, 2, ---$ Similarly, the n^{th} forward differences are defined by the formula

$$\Delta^n y_r = \Delta^{n-1} y_{r+1} - \Delta^{n-1} y_r, \quad r = 0, 1, 2 ---$$

Backward differences:

An mentioned earlier let $y_0, y_1, y_2 --- y_r$ be the values of a function $y = f(x)$ corresponding to the values $x_0, x_1 --- x_r ---$ of x respectively then $\nabla y_1 = y_1 - y_0$, $\nabla y_2 = y_2 - y_1$ --- are called the first backward differences.

In general, $\nabla y_r = y_r - y_{r-1}$, $r = 1, 2, 3 ---$

The symbol ∇ is called the backward difference operator. Like the operator Δ, this operator is also a linear operator. It is also called nable.

The differences of the 1st backward differences are called second backward differences and are denoted by $\nabla^2 y_2, \nabla^3 y_3 \ldots \nabla^2 y_r \ldots$

i.e, $\nabla^2 y_2 = \nabla y_2 - \nabla y_1, \nabla^3 y_3 = \nabla y_3 - \nabla y_2 \ldots$

In general $\nabla^2 y_r = \nabla y_r - \nabla y_{r-1}$ $[\nabla^3 y_3 = \nabla y_3 - \nabla y_2]$; $r = 2, 3 \ldots$

Illy, the nth backward differences are defined by the formula $\nabla^n y_r = \nabla^{n-1} y_r - \nabla^{n-1} y_{r-1}$.

Illy the nth backward differences are defined by the formula

$$\nabla^n y_r = \nabla^{n-1} y_r - \nabla^{n-1} y_{r-1}, \quad r = n, n+1 \ldots$$

The symbol ∇^n is referred to as the nth backward difference operator.

Newton's forward Interpolation formula:-

Let $y = f(x)$ be a polynomial of nth degree and taken in the following formula.

$$y = f(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + \ldots + a_n(x-x_0) \ldots$$

from this we get newton's forward interpolation.

i.e, $$y = f(x) = f(x_0 + ph) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \ldots + \frac{p(p-1)\ldots(p-n+1)}{n!}\Delta^n y_0$$

where $p = \frac{x - x_0}{h}$

Newton's backward interpolation formula:-

If we consider $y_n(x) = a_0 + a_1(x-x_n) + a_2(x-x_n)(x-x_{n-1}) + \ldots + a_n(x-x_n)(x_n - x_{n-1}) \ldots (x-x_0)$

from this we can get newton's backward interpolation formula.

i.e, $y_n(x) = y_n + P\nabla y_n + \frac{P(P+1)}{2!}\nabla^2 y_{n-1} + \dots + \frac{P(P+1)\dots(P+n-1)}{n!}$

$\nabla^n y_n$ where $p = \frac{x - x_n}{n}$.

Important questions:

1. construct a forward difference table from the following data.

x	0	1	2	3	4
y_x	1	1.5	2.2	3.1	4.6

2. Evaluate (i) $\Delta \cos x$ (ii) $\Delta \log f(x)$ (iii) $\Delta^2 \sin(px+q)$ (iv) $\Delta \tan^{-1} x$ & (V) $\Delta^n e^{ax+b}$

3. P.T $1 + \mu^2 \delta^2 = (1 + \frac{1}{2}\delta^2)^2$

4. using the method of separation of symbols S.T $\Delta^n u_{x-n} = u_x - n u_{x-1} + \frac{n(n-1)}{2} u_{x-2} + \dots + (-1)^n u_{x-n}$

5. The following table gives a set of values of x and the corresponding values of $y = f(x)$

x	10	15	20	25	30	35
y	19.97	21.51	22.47	23.52	24.65	25.89

6. use newton's forward formula to evaluate $f(1.6)$ from the following data.

x	1	1.4	1.8	2.2
$f(x)$	3.49	4.82	5.92	6.5

7. using Newton's ~~for~~ backward formula find $f(0.7)$ from the following data.

x	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)=y$	2.68	3.04	3.33	3.68	3.96	4.21

8. From the following table find y and $x=38$.

x	30	35	40	45	50
y	15.9	14.9	14.1	13.3	12.5

Course/Group:	M.SC Physics
Paper:	mathematical methods of physics
Name of the Topic	complex variables
Hours required	15
Learning Objectives	Concepts of Analytic functions, Taylor's theorem, Cauchy-Riemann Conditions.
Previous knowledge to be reminded	
Topic Synopsis	Function of complex Number - definition - properties Analytic function Cauchy - Riemann conditions *(Continue on the reverse side if needed)* Cauchy's integral theorem Taylor's theorem Laurent's theorem
Examples/Illustrations	Given on the Reverse side
Additional inputs	
Teaching Aids used	Black Board
References cited	mathematical Physics - Satya Prakash, S. Chand
Student Activity planned after the teaching	Assignments and seminars
Activity planned outside the Class room, if any	
Any other activity	

Signature of the Lecturer

Complex Number:-

A Number which is of the form $a+ib$ where $i=\sqrt{(-1)}$ and a, b are real Numbers is called a complex Number

we defined a complex number as an ordered pair of real numbers like (x, y) (or) (a, b). If we write the complex number

$z=(x,y)=x+iy$ where x, y are reals then x is called the real and 'y' is the imaginary part of the complex number of z

usually the real part x is denoted by $R(z)$ (or) $Re(z)$ and imaginary part 'y' is denoted by $I(z)$ (or) $Img(z)$

Two complex numbers are said to be Equal if and only if three real parts are Equal and their imaginary parts are Equal

For Example,

$$(x_1, y_1) = (x_2, y_2) \Leftrightarrow x_1 = x_2 \text{ and } y_1 = y_2$$

Complex Conjugate:

The complex conjugate or simply the conjugate of a complex number $z=(x,y)=x+iy$ is defined as a complex number $x-iy$ and is denoted by $\bar{z}$ (or) z^*

Properties of modulus:-

1. $|z_1+z_2| \leq |z_1|+|z_2|$ 2. $|z_1-z_2| \geq |z_1|-|z_2|$

3. $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$

* properties of arguments

1. $\arg(z_1 z_2 \cdots z_n) = \arg z_1 + \arg z_2 + \cdots + \arg z_n$

2. $\arg(z_1/z_2) = \arg z_1 - \arg z_2$.

* set of points: Any collection of points in a complex plane is called an aggregate or a set of points

* Neighbourhood of a point:

A Neighbourhood of a point z_0 in the Argand plane is the set of all points z. such that $|z - z_0| < \varepsilon$, where ε is any arbitrarily small positive number choosen

* Limit point:-

A point z_0 is said to be the limit point of the set of points s. in the Argand plane. If every neighbourhood of z_0. contains points of the set s other than z_0.

* Interior point:-

A limit point z_0 of the set s. is said to be an interior point if the neighbourhood of z_0, there exists entirely the points (or) the set s.

* Boundary point:-

A limit point z_0. which is not an interior point is said to be the boundary point

* closed set:-

If all the limit point of a set belong to the set, then the set is said to be closed set.

Important questions:

1. The necessary and sufficient condition for a complex valued function to be analytic is $U_x = V_y$ and $V_x = -U_y$

i.e $\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$ (CR eqn)

2. Prove that $u = e^{-x}(x \sin y - y \cos y)$ is harmonic and also find the value of v

3. State and prove Cauchy's Integral theorem

4. Using Cauchy's Integral theorem, Evaluate the integral $\oint \frac{dz}{z}$, where C is simple closed curve

5. State and prove Liouville's theorem

6. State and prove Taylor's series theorem

7. State and prove Laurents theorem.

8. State and prove Cauchy Residue theorem.

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TEACHIG NOTES
2019-2020

Name of the Department / Subject : MATHEMATICS

Name of the Lecturer : N. L. N. MALLIKA

Course/Group:	I B.SC
Paper:	Paper-I
Name of the Topic	Orthogonal trajectories, with polar-co-ordinates, DE's of first order but not first degree
Hours required	
Learning Objectives	Solve orthogonal trajectories & O.T of DE's with polar co-ordinates, solve the first order but not first degree diff-Eq's.
Previous knowledge to be reminded	Properties of differentiations & Integrations.
Topic Synopsis	Orthogonal trajectories, Self orthogonal trajectories polar-coordinates. Form of DE's of first degree and higher degree and solvable **(Continue on the reverse side if needed)** for p, x, y and problems, Clairaut's Equations.
Examples/Illustrations	Given on the back side.
Additional inputs	2 CT
Teaching Aids used	Blackboard
References cited	S. Chand (V. Venkateswara Rao, N. Krishnamurthy, B.V.S.S Sarma, S. Anjaneya Sastry & S. Ranganatham)
Student Activity planned after the teaching	Seminars & Assignments
Activity planned outside the Class room, if any	
Any other activity	

Signature of the Lecturer

Trajectory :- If a curve 'C' cuts every member of a given family of curves 'T' according to some specified law, then the curve 'C' is called 'trajectory' of the given family of curves T.

Orthogonal trajectories :-

If a curve C cuts every member of a given family of curves T, at a right angle is called a orthogonal trajectories of family T.

Polar co-ordinates :-

If $f(r,\theta,c)=0$, C is being a parameter, is the polar Eq. of the family of curves then the D.E of the family of the orthogonal trajectories is $F(r,\theta,-r^2\frac{d\theta}{dr})=0$.

First order D.E but not first degree :-

An Eq. of the form $\left(\frac{dy}{dx}\right)^n+P_1\left(\frac{dy}{dx}\right)^{n-1}+\dots+P_{n-1}\left(\frac{dy}{dx}\right)+P_n=0$ is called first order n^{th} degree diff. eq's where $P_1, P_2, \dots P_n$ are functions of 'x' and 'y' where $p=\frac{dy}{dx}$ in Eq. ① can be written as $p^n+P_1p^{n-1}+P_2p^{n-2}+\dots+P_{n-1}p+P_n=0$.

Eq ① can also be written as $f(x,y,p)=0$.

Methods of finding a general solution of $F(x,y,p)=0$.

Method - I :- Solvable for 'p' in terms of x and y :-

let $f(x,y,p)=0$ be the n^{th} degree polynomial then $F(x,y,p)=[p-f_1(x,y)][p-f_2(x,y)]\dots[p-f_n(x,y)]$.

If $\phi(x, y, c_i) = 0$ is a solution of $p - f_i(x,y) = 0$

for $i = 1, 2, \ldots n$ then $\phi(x,y,c_1) \cdot \phi(x,y,c_2) \ldots \phi(x,y,c_n) = 0$

is the solution of $F(x,y,p) = 0$.

Method 2 :- Solvable for 'x' :-

If Given D.E $f(x,y,p) = 0$ is of first degree in 'x' then it is solvable for 'x' and it can be written as a function of y and p.

i.e, $x = F(y,p) \longrightarrow (1)$

Diff. w.r.t. 'y'.

$\Rightarrow \frac{dx}{dy} = g\left(y, p, \frac{dp}{dy}\right)$ which is a first order D.E in 'y' and 'p'.

$\Rightarrow$ Solution of above D.E is $\phi(y,p,c) = 0$ $\longrightarrow (2)$

Eliminating 'p' from (1) & (2) we get a relation between x, y & c which is required G.S.

Method 3 :- Solvable for 'y' :-

Suppose $f(x,y,p) = 0$ is of first degree in 'y' then it is solvable for 'y' and it can be expressed as a function of x and p. i.e, $y = F(x,p) \longrightarrow (1)$

diff. w.r.t. 'x'.

$\Rightarrow \frac{dy}{dx} = g\left(x, p, \frac{dp}{dx}\right)$ which is first order D.E in x & p.

$\Rightarrow$ Solution for this Eq. is $\phi(x,p,c) = 0$. $\longrightarrow (2)$

Eliminating 'p' from Eq. (1) & (2) we get a relation between x, y & c which is required general solution.

Clairaut's Equation :- The D.E of the form $y = xp + f(p)$ is called Clairaut's Eq. and G.S of Clairaut's Eq. is $y = xc + f(c)$.

Important Questions

1. Find the orthogonal trajectories of the family of curves $y = \frac{1}{\log c_1 x}$ where c_1 is parameter.
2. Find the orthogonal trajectories of the family of curves $x^{2/3} + y^{2/3} = a^{2/3}$ where 'a' is parameter.
3. Find the orthogonal trajectories of family of Co-axial of circles $x^2+y^2+2gx+c=0$, where 'g' is a parameter
4. S.T family of confocal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ is self orthogonal, where 'λ' is a parameter.
5. Find the orthogonal trajectories of the family of curves $r = a(1-\cos\theta)$ where 'a' is a parameter.
6. Find the orthogonal trajectories of family of curves $r = \frac{2a}{1+\cos\theta}$, where 'a' is parameter.
7. Solve $p^2+2py\cot x = y^2$.
8. Solve $x^2\left(\frac{dy}{dx}\right)^2 - 2xy\frac{dy}{dx} + 2y^2 - x^2 = 0$.
9. Solve $x^2\left(\frac{dy}{dx}\right)^2 + xy\left(\frac{dy}{dx}\right) - 6y^2 = 0$.
10. Solve $y^2 \log y = xpy + p^2$.
11. Solve $2px = 2\tan y + p^3\cos^2 y$.
12. Solve $y + px = p^2x^4$
13. Solve $y = 2px + x^2p^4$.
14. $y = xp^2 + p$
15. Solve $(y-px)(p-1) = p$.
16. Solve $(py+x)(px-y) = 2p$.

Course/Group:	B.Sc
Paper:	Paper - III
Name of the Topic	Subgroups, Cosets & Lagrange's theorem
Hours required	08
Learning Objectives	Subgroups, Union & Intersection concepts of subgroups, Proving how group will be a subgroup, Concepts of Coset & Normaliser of an element
Previous knowledge to be reminded	Properties of groups
Topic Synopsis	Complex definition, subgroups, criterion for the product of two subgroups to be a subgroup, union and intersection (Continue on the reverse side if needed) of subgroups, Definition of Coset, Properties of Cosets, Normaliser of an element of a group.
Examples/Illustrations	Given on the reverse side
Additional inputs	ICT
Teaching Aids used	Black board
References cited	S. Chand
Student Activity planned after the teaching	Seminars, Assignment
Activity planned outside the Class room, if any	
Any other activity	

N. H. N. Harika

Signature of the Lecturer

Complex definition:- Any subset of a group G is called a complex of G.

Eg. The set of integers is a complex of group $(R, +)$.

Multiplication of two complexes :-

If M & N are any two complexes of a group G then $MN = \{mn \in G / m \in M, n \in N\}$.

Clearly $MN \subseteq G$ & MN is called the product of the complexes M, N of G.

Subgroups :- Let $(G, \cdot)$ be a group. Let H be a non-empty subset of G such that $(H, \cdot)$ be a group. Then H is called a subgroup of G.

It is denoted by $H \leq G$ or $G \geq H$ and $H < G$ or $G < H$ we mean $H \leq G$ but $H \neq G$.

Union and intersection of subgroups :-

If H_1 & H_2 are two subgroups of a group G then $H_1 \cap H_2$ is also a subgroup of G.

Cosets :-

Let $(H, \cdot)$ be a subgroup of the group $(G, \cdot)$. Let $a \in G$ then the set $aH = \{ah / h \in H\}$ is called a left coset of H in G generated by 'a' and the set

$Ha = \{ha / h \in H\}$ is called a right coset of H in G generated by 'a'. Also aH, Ha are called cosets of H generated by 'a' in G.

Congruence modulo H :-

Let $(G, \cdot)$ be a group and $(H, \cdot)$ be a subgroup of G. For $a, b \in G$ if $b^{-1}a \in H$ we say that $a \equiv b \pmod{H}$.

Normalizer of an element of a group :-

If 'a' is an element of a group G, then the normalizer of 'a' in G is the set of all these elements of G which commute with 'a'. The normalizer of 'a' in G is denoted by $N(a)$, where $N(a) = \{x \in G \mid ax = xa\}$

The normalizer $N(a)$ is a subgroup of G.

Self-conjugate element of a group :-

$(G, \cdot)$ is a group and $a \in G$ such that $a = x^{-1}ax$, $\forall x \in G$. Then 'a' is called self conjugate element of G. A self-conjugate element is sometimes called an invariant element.

Imp. Questions

1. If H is any subgroup of a group G, then $H^{-1} = H$.
2. H is a non-empty complex of a group G. The necessary and sufficient condition for H, to be a subgroup of G is $a, b \in H \Rightarrow ab^{-1} \in H$ where b^{-1} is the inverse of b in G.
3. The necessary and sufficient condition for a finite complex H of a group G to be a subgroup of G is $a, b \in H \Rightarrow ab \in H$.
4. If H and K are two subgroups of a group G, then HK is a subgroup of G iff $HK = KH$.
5. If H_1 & H_2 are two subgroups of a group G then $H_1 \cap H_2$ is also a subgroup of G.
6. The union of two subgroups of a group is a subgroup iff one is contained in the other.
7. If a, b are any two elements of a group $(G, \cdot)$ and H any subgroup of G then $Ha = Hb \Leftrightarrow ab^{-1} \in H$ and $aH = bH \Leftrightarrow a^{-1}b \in H$.
8. Any two left (right) cosets of a subgroup are either disjoint or identical.
9. State and prove Lagrange's theorem.

Course/Group:	II B-SC
Paper:	Paper-III
Name of the Topic	~~Right line~~ Spheres
Hours required	18+1=19
Learning Objectives	Find the eq. of Spheres and Properties of spheres.
Previous knowledge to be reminded	Properties of Straight lines.
Topic Synopsis	Def. and eq. of the sphere, eq. of the sphere through four given points, plane sections of a sphere, Intersection **(Continue on the reverse side if needed)** of two spheres, eq. of a circle, sphere through a given circle, Intersection of a sphere and line, power of a point, Tangent plane, plane of contact, polar plane, pole of a plane, conjugate points, conjugate planes.
Examples/Illustrations	Given on reverse side
Additional inputs	ICT
Teaching Aids used	Black boards
References cited	S. chand.
Student Activity planned after the teaching	Assignments, seminars.
Activity planned outside the Class room, if any	
Any other activity	

Signature of the Lecturer

Sphere: The set of points in a sphere which are at a constant distance $a\ (\geq 0)$ from a fixed point C is called a sphere.

→ Eq. to the sphere with centre (x_1, y_1, z_1) and radius a is $(x-x_1)^2+(y-y_1)^2+(z-z_1)^2 = a^2$

→ Eq. to the sphere is of the form
$x^2+y^2+z^2+2ux+2vy+2wz+d=0$ where
centre $C = (-u, -v, -w)$ & radius $r = \sqrt{u^2+v^2+w^2-d}$

Concentric spheres:-

Spheres with the same centre are known as concentric spheres.

→ A plane section of a sphere is a circle.

Great circle:-

If a plane π passes through the centre of a sphere S, then the plane section of the sphere is called a great circle.

→ If two spheres intersect then the locus of the set of all points of intersection is a circle.

→ If a plane $U=0$ intersects the sphere $S=0$, in a circle C, then for all real values of λ, $S+\lambda U=0$ represents the equation to a sphere passing through the circle C.

→ length of the tangent line from (x_1, y_1, z_1) to the sphere $S=0$ is $\sqrt{S_{11}}$.

→ Eq. to the plane of Contact of all points (x_1, y_1, z_1) w.r.t. the sphere $S=0$ of non-zero radius S is $S_1=0$.

→ The Eq. of the polar plane of the point (x_1, y_1, z_1) w.r.t. the sphere $S=0$ is $S_1=0$.

→ If $x^2+y^2+z^2-a^2=0$ is a sphere, then the pole of the plane $lx+my+nz=p$ is $\left(\frac{a^2 l}{p}, \frac{a^2 m}{p}, \frac{a^2 n}{p}\right)$.

Conjugate points, Conjugate planes :-

If S is a sphere, A, B are two points such that the polar plane of B w.r.t. S passes through A then A, B are called Conjugate points w.r.t. S.

→ The polar planes of A & B are called Conjugate planes.

→ L, L' are two lines such that the polar plane of every point on L w.r.t. sphere S, passes through L'. Then L, L' are called Conjugate lines or polar lines.

Important Questions

1. Find the Eq. to the sphere through $O=(0,0,0)$ and making intercepts a, b, c on the axes.

2. A plane passes through a fixed point (a,b,c) and intersects the axes in A, B, C. S.T the centre of the sphere $OABC$ lies on $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$.

3. A sphere of constant radius 'k' passes through the origin and intersects the axes in A, B, C. P.T the centroid of ΔABC lies on the sphere $9(x^2+y^2+z^2) = 4k^2$.

4. S.T the two circles $x^2+y^2+z^2-y+2z=0$, $x-y+z=2 \rightarrow (1)$ $x^2+y^2+z^2+x-3y+z-5=0$, $2x-y+4z-1=0$ lies on the same sphere and find its Equation.

5. Find the Eq. of the sphere for which the circle $x^2+y^2+z^2+7y-2z+2=0$, $2x+3y+4z=8$ is a great circle.

6. S.T the plane $2x-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ and find the points of contact.

7. Find the pole of the plane $x-y+5z-3=0$ w.r.t the sphere $x^2+y^2+z^2=9$.

Course/Group:	II B.Sc
Paper:	Paper-II
Name of the Topic	Real numbers, Real sequences
Hours required	20
Learning Objectives	Concepts of Inf, Sup, upper bound, lower bound, limits, Range, convergence.
Previous knowledge to be reminded	Properties of groups.
Topic Synopsis	The algebraic and order properties of R, Absolute value and Real line, Completeness Property of R, Applications of supremum Property Intervals sequences and their (Continue on the reverse side if needed) limits, Range & boundedness of sequences, limit of a sequences and convergent sequences, subsequences, Monotone sequences.
Examples/Illustrations	Given on the reverse side
Additional inputs	ICT
Teaching Aids used	Black board
References cited	S. Chand, Golden series
Student Activity planned after the teaching	Seminars, assignment.
Activity planned outside the Class room, if any	
Any other activity	

N. A. N. [signature]
Signature of the Lecturer

Field axioms :-

The set of real numbers R is a field with two binary operations called addition (+) and multiplication (·) which satisfy the following axioms $A_1, A_2, A_3, A_4, M_1, M_2, M_3, M_4$ & D.

A_1 : $a+b=b+a, \forall a,b \in R$

A_2 : $(a+b)+c = a+(b+c), \forall a,b,c, \in R$.

A_3 : R contains a unique element '0' $\ni$ $a+0=a, \forall a \in R$.

A_4 : For any $a \in R$ there corresponds a unique element $-a \in R$ such that $a+(-a)=0$, "$-a$" is called negative of a or additive inverse. similarly M_1, M_2, M_3, M_4.

The order Axioms :-

The order relation '>' defined for pairs of real numbers satisfies the following axioms.

O_1 : for $a,b \in R$ one and only one of the following is true. $a>b, a=b, b>a$ (Trichonomy).

O_2 : For $a,b,c \in R$, $a>b, b>c \Rightarrow a>c$ (Transitivity)

O_3 : For $a,b,c \in R, a>b \Rightarrow a+c=b+c$ (Monotone property)

O_4 : For $a,b,c \in R$, $a>b, c>0 \Rightarrow ac>bc$ (Monotone Property for multiplication)

The relation '>' satisfies the above four axioms is called a linear order & R is called linearly ordered field.

Boundedness of subsets of R:-

An aggregate S is said to be bounded above, if $\exists\ k_1 \in R \ni x \in S \Rightarrow x \le k_1$. The number k_1 is called an upper bound.

If k_1 is an upper bound of S then any number greater than k_1 is also an upperbound of S.

Least upper bound (l.u.b) or Supremum:-

If 'u' is an upper bound of an aggregate S and any real number less than u is not an upper bound of S, Then u is called least upper bound or Supremum of S.

Lower bound:-

An aggregate S is said to be bounded of an aggregate S and any real number greater than V is not a lower bound of S, Then V is called greatest lower bound or Infimum of S.

Boundedness:- An aggregate S is said to be bounded if it is both bounded below and bounded above.

Subsequences:- If $\{s_n\}$ is a sequence and $\{n_k\}$ is a sequences of positive integers such that $n_1 < n_2 < \ldots$. then the sequence $\{s_{n_k}\}$ is called subsequence of $\{s_n\}$.

Range of Sequence:- The set of all terms of a sequences is called range set or range of the sequence. Range of set of $\{s_n\} = \{s_n / n \in Z^+\}$

Convergent sequence:- A sequence $\{s_n\}$ is said to be convergent if $\exists\ l \in R \ni$ for each $\epsilon > 0\ \exists\ m \in Z^+ \ni$ $|s_n - l| < \epsilon,\ \forall n \ge m$.

Monotone Sequence :- A sequence $\{S_n\}$ which is either increasing or decreasing is called Monotone sequence.

Important Questions

1. A sequence can have atmost one limit (or) a convergence sequence has unique limit.
2. State and prove Sandwich theorem (or) Squeeze theorem.
3. A monotone sequence is convergent iff it is bounded.
4. Prove that the seq. $\{S_n\}$ where $S_n = \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n+n}$ is convergent.
5. Prove that the seq $\{S_n\}$ where $S_n = \frac{3n+4}{2n+1}$ is decreasing and bounded below.
6. State and prove Bolzano-weierstrass theorem.
7. If the sequence $\{S_n\}$ is convergent, then $\{S_n\}$ is a Cauchy sequence.
8. State and prove Cauchy general principal of convergence (or) Cauchy's theorem.
9. If $\{S_n\}$ is a Cauchy sequence then $\{S_n\}$ is convergent
10. State and prove Cauchy's first theorem on limits.

Course/Group:	I BCA
Paper:	Elementary mathematics.
Name of the Topic	Numerical Methods.
Hours required	10.
Learning Objectives	Roots finding, algebraic equations solving with different types of root finding methods
Previous knowledge to be reminded	Polynomials, real roots, roots of a polynomials.
Topic Synopsis	Solution of algebraic & transcendental Equations, Bisection method, method of (Continue on the reverse side if needed) false position, Newton - Raphson method.
Examples/Illustrations	Given on Reverse side.
Additional inputs	ICT, PPT
Teaching Aids used	Black board
References cited	S. Chand (Mathematical methods)
Student Activity planned after the teaching	Seminars, Assignments.
Activity planned outside the Class room, if any	
Any other activity	

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Signature of the Lecturer

Polynomial function :- A function $f(x)$ is said to be a polynomial function if $f(x)$ is a polynomial in x.
i.e. $f(x) = a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n$ where $a_0 \neq 0$, the co-efficients $a_0, a_1, \dots a_n$ are real constant and n is a non-negative integer.

Algebraic function A function which is sum or difference or product of two polynomials is called an algebraic function. otherwise the function is called transcendental function.

If $f(x)$ is algebraic function, then $f(x)=0$ is called an algebraic equation.

If $f(x)$ is transcendental function, then $f(x)=0$ is called a transcendental equation.

Eg: $f(x) = c_1e^x + c_2e^{-x} = 0$, $f(x) = 2\log x - \frac{\pi}{4} = 0$ are examples of transcendental equations.

Bisection Method :-

Suppose we know an equation of the form $f(x)=0$ has exactly one real root between two real numbers x_0, x_1.

* The number chosen so that $f(x_0)$ & $f(x_1)$ will have opposite sign.
* Let us bisect $[x_0, x_1]$ into two half intervals & find the midpoint $x_2 = \frac{x_0+x_1}{2}$.
* If $f(x_2)=0$ then x_2 is a root.

* If $f(x_1)$ & $f(x_2)$ have same sign then root lies between x_0 & x_2, The interval taken as $[x_0, x_2]$. otherwise the root lies in the root lies in the interval $[x_2, x_1]$.

Regula - falsi method :-

* In Regula falsi position method, we will find the root of the equation $f(x)=0$.
* Consider two initial approximate values x_0 & x_1 near the required root so that $f(x_0)$ & $f(x_1)$ have different signs.

$\Rightarrow$ root lies between x_0 & x_1.

Let $A=(x_0, f(x_0))$ and $B=(x_1, f(x_1))$ be the points on the curve $y=f(x)$, Equation AB is $\frac{y-f(x_0)}{x-x_0} = \frac{f(x_1)-f(x_0)}{x_1-x_0}$ → ①

$$\Rightarrow x = x_0 - \frac{x_1-x_0}{f(x_1)-f(x_0)} \cdot f(x_0) \rightarrow ②$$

which is an approximated root, when the interval in which it lies is small. If the new value of x is taken as x_2 then ② becomes,

$$x_2 = x_0 - \frac{(x_1-x_0)}{f(x_1)-f(x_0)} f(x_0)$$

$$= \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1)-f(x_0)}$$

Continue this procedure till the root is found to the desired accuracy.

i.e $x_{i+1} = \frac{x_{i-1} f(x_i) - x_i f(x_{i-1})}{f(x_i) - f(x_{i-1})}$.

Newton - Raphson Method :

Let x_0 be an approximate root of $f(x)=0$, & let $x_1 = x_0 + h$ be correct root which implies that $f(x_1)=0$ we use Taylor's theorem & expand

$$f(x_1) = f(x_0+h) = 0.$$

$$f(x_0+h) = f(x_0) + h f'(x_0) + h^2 f''(x_0) + \cdots$$

$\Rightarrow f(x_0) + h f'(x_0) = 0$

$\Rightarrow h = -\dfrac{f(x_0)}{f'(x_0)}$

$\Rightarrow$ Substituting this in x_1, we set $x_1 = x_0 + h$

$\Rightarrow x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

x_1 is a better approximation than x_0.

Successive approximations are given by $x_2, x_3, \dots x_{n+1}$

where $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}$

This is called Newton-Rapheon formula.

Important Questions

1. Find a positive root of $x^3 - x - 1 = 0$ (correct to two decimal places) by bisection method.
2. Find out the square root of 25 given $x_0 = 2.0, x_1 = 7.0$ using Bisection method.
3. Find a real root of $x \log_{10} x = 1.2$ which lies between 2 and 3 by bisection method & from also using Regula false method.
4. Find a real root of $xe^x = 3$ using Regula-Falsi method.
5. Find the positive root of $f(x) = x^3 - 2x - 5 = 0$.
6. Find the square root of 10 by using Newton-Rapheon method
7. Using the Newton-Rapheon method, find the root of $f(x) = e^x - 3x$ that lies between 0 & 1.
8. Find a real root of the equation $xe^x - c8x = 0$ using Newton Rapheon method.

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TEACHIG NOTES
2019-2020

Name of the Department / Subject : Mathematics

Name of the Lecturer : G.T. swarnalata

Course/Group:	I B. Sc
Paper:	Paper - I
Name of the Topic	orthogonal trajectories & polar coordinates D.E of first order but not first degree
Hours required	16
Learning Objectives	solve orthogonal trajectories & polar coordinates. solve the first order but not first degree differential equations.
Previous knowledge to be reminded	Properties of differentials & integrals find the roots.
Topic Synopsis	orthogonal trajectories, self orthogonal trajectories, polar coordinates form of D.E of first degree & higher degree **(Continue on the reverse side if needed)** and solvable for p, x, y and problems on clairaut's equation
Examples/Illustrations	Given on the reverse side
Additional inputs	ICT
Teaching Aids used	Black board
References cited	S. chand
Student Activity planned after the teaching	Seminars, assignment
Activity planned outside the Class room, if any	
Any other activity	

G.T. [illegible]

Signature of the Lecturer

Trajectory:-

If a curve 'c' cuts every member of a given family of curves 'T' according to some specified law, then the curve 'c' is called trajectory of the given family of curves "T".

Orthogonal Trajectory:-

If a curve 'c' cuts every member of a given family of curves "T" at a right angle, then the curve 'c' is called an orthogonal trajectory of the family T.

Self orthogonal family of curves:-

If each member of a given family of curves cuts every other member of the family at right angle, then the given family of curves is said to be self orthogonal.

Polar coordinate:-

If $f(r, \theta, c) = 0$, 'c' being the parameter is the polar equation of the family of curves, then the differential equation of the family of the orthogonal trajectories is $f\left(r, \theta, -r\frac{d\theta}{dr}\right) = 0$.

Methods of finding a general solution of $F(x,y,p)=0$

Method-I: Solvable for p in terms of x and y:-

Let $f(x,y,p)=0$ be the nth degree polynomial then $F(x,y,p)=[p-f_1(x,y)][p-f_2(x,y)]\cdots[p-f_n(x,y)]$.

If $\phi(x,y,c_i)=0$ is a solution of $p-f_i(x,y)=0$ for $i=1,2,\cdots n$ then $\phi(x,y,c_1),\phi(x,y,c_2)\cdots\phi(x,y,c_n)=0$ in the sol. of $F(x,y,p)$.

Method-2:- Solvable for y:-

Suppose $f(x,y,p)=0$ is the given D.E & that it is possible to find y in terms of x and p $\Rightarrow y=f(x,p) \rightarrow ①$

$\Rightarrow p=\frac{\partial f}{\partial x}+\frac{\partial f}{\partial p}\cdot\frac{\partial p}{\partial x} \rightarrow ②$ where $p=\frac{dy}{dx}$

which is a first order D.E in p and x & its solution and of the form $\phi(x,p,c)=0 \rightarrow ③$

eliminating p from equ ① & ② we get a solution x,y & c which is required general solution.

Method-3:- Solvable for x:-

If 'x' is not of first degree is the given DE $f(x,y,p)=0$ then it is solvable for x and it can be written as a functions of y and p.

$x=f(y,p) \rightarrow ①$ Diff. w.r.t. y we have

$\frac{dx}{dy}=\frac{\partial f}{\partial y}+\frac{\partial f}{\partial p}\cdot\frac{\partial p}{\partial y} \Rightarrow \frac{1}{p}=\frac{\partial f}{\partial y}+\frac{\partial f}{\partial p}\cdot\frac{\partial p}{\partial y}$

which is a first order D.E is p and y and its sol. is of the form $\phi(y,p,c)=0$ ②

eliminating p from ① & ② we get a relation b/w x,y & c which is the required general solution.

Important Questions

1) Find the orthogonal trajectories of the family of curves $y = \frac{1}{\log c_1 x}$ where c_1 is parameter.

2) Find the orthogonal trajectories of the family of curves $x^{2/3} + y^{2/3} = a^{2/3}$ where 'a' is parameter.

3) Find the orthogonal trajectories of the family of coaxial of circles $x^2+y^2+2gx+C=0$ where g is parameter.

4) S.T. family of confocal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ is self orthogonal, where λ is parameter.

5) Find the orthogonal trajectories of the family of curves $r = a(1-\cos\theta)$ where 'a' is parameter.

6) Find the orthogonal trajectories of the family of curves $r = \frac{2a}{1+\cos\theta}$, where 'a' is parameter.

(7) solve $p^2 + 2py\cot x = y^2$

8) solve $x^2\left(\frac{dy}{dx}\right)^2 - 2xy\frac{dy}{dx} + 2y^2 - x^2 = 0$

9) solve $x^2\left(\frac{dy}{dx}\right)^2 + xy\frac{dy}{dx} - 6y^2 = 0$

10) solve $y^2\log y = xpy + p^2$

11) solve $2px = 2\tan y + p^3\cos^2 y$

12) solve $y + px = p^2x^4$

13) solve $y = 2px + x^2p^4$

14) solve $y = xp^2 + p$

15) solve $(y - xp)(p-1) = p$

16) solve $(py+x)(px-y) = 2p$.

Course/Group: I B.Sc	
Paper: Paper-II	
Name of the Topic	Spheres
Hours required	17.
Learning Objectives	Find the equ. of spheres and properties of spheres
Previous knowledge to be reminded	properties of st. lines
Topic Synopsis	Def & equ. to the sphere, equ. of the sphere through four given points, plane section of a sphere, intersection **(Continue on the reverse side if needed)** of equ. of circle, intersection of a sphere & line, power of a point, Tangent plane, plane of contact, pole of planes
Examples/Illustrations	Given on reverse side
Additional inputs	ICT
Teaching Aids used	Blackboard
References cited	S.chand
Student Activity planned after the teaching	Seminar, assignment
Activity planned outside the Class room, if any	
Any other activity	

G.T.S. [signature]

Signature of the Lecturer

Spheres:-

The set of points in a space which are at a constant distance $a (>0)$ from a fixed point 'c' is called a sphere.

→ equation to the sphere with centre (x_1, y_1, z_1) and radius 'a' is $(x-x_1)^2+(y-y_1)^2+(z-z_1)^2=a^2$

where ⇒ $x^2+y^2+z^2+2ux+2vy+2wz+d=0$

where centre $c=(-u,-v,-w)$ & radius $=\sqrt{u^2+v^2+w^2-d}$

Concentric sphere:-

Sphere with the same centre are known as concentric sphere.

→ A plane section of a sphere is a circle

Great circle:-

If a plane π passes through the centre of a sphere S, then the plane section of the sphere is called great circle.

→ If two spheres intersect, then the locus of the set of all points of intersection is a circle.

→ length of the tangent line from (x_1, y_1, z_1) to the sphere $S=0$ is $\sqrt{S_{11}}$

→ equ. to the plane of contact of all points (x_1, y_1, z_1) w.r.t. the sphere $S=0$ of non-zero radius S is $S_1=0$

→ The equ. of the polar plane of the point (x_1, y_1, z_1) w.r.t. the sphere $S=0$ is $S_1=0$.

→ If $x^2+y^2+z^2-a^2=0$ is a sphere, then the pole of the plane $lx+my+nz=p$ is $\left(\frac{a^2 l}{p}, \frac{a^2 m}{p}, \frac{a^2 n}{p}\right)$.

<u>conjugate points</u>, <u>conjugate planes</u>:-

If 'S' is a sphere. A, B are two points such that the polar plane of B w.r.t. spheres through A then A, B are called conjugate points w.r.t. S.

→ The polar plane of A & B are called Conjugate planes

→ L, L' are two lines such that the polar plane of every point on L w.r.t. spheres S, passes through L', then L, L' are called conjugate lines or polar lines.

Important Questions

1) Find the equ. to the sphere through $0(0,0,0)$ and making intercepts a, b, c on the axis.

2) A plane passes through a fixed point (a, b, c) and intercepts the axes in A, B, C. S.T. the centre of the sphere $OABC$ lies on $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$

3) A sphere of constant radius k passes through the origin and intercepts the axes in A, B, C. P.T. the centroid of ΔABC lies on the sphere $9(x^2+y^2+z^2)=4k^2$.

4) S.T. the two circles $x^2+y^2+z^2-y+2z=0$, $x-y+z=2$, $x^2+y^2+z^2+x-3y+z-5=0$, $2x-y+4z-1=0$ lies on the same sphere & find its equ.

5) Find the equ. of the sphere for which the circle $x^2+y^2+z^2+7y-2z+2=0$, $2x+3y+4z=8$ is a great circle.

6) S.T. the plane $2x-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ & find the point of contact.

Important Questions

1) Find the equ. to the sphere through $O(0,0,0)$ and making intercepts a, b, c on the axis

2) A plane passes through a fixed point (a,b,c) and intercepts the axes in A, B, C. S.T. the centre of the sphere $OABC$ lies on $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$

3) A sphere of constant radius k passes through the origin and intercepts the axes in A, B, C. P.T. the centroid of ΔABC lies on the sphere $9(x^2+y^2+z^2)=4k^2$.

4) S.T. the two circles $x^2+y^2+z^2-y+2z=0$, $x-y+z=2$, $x^2+y^2+z^2+x-3y+z-5=0$, $2x-y+4z-1=0$ lies on the same sphere & find its equ.

5) Find the equ. of the sphere for which the circle $x^2+y^2+z^2+7y-2z+2=0$, $2x+3y+4z=8$ is a great circle.

6) S.T. the plane $2x-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ & find the point of contact.

Important Questions

1) Find the equ. to the sphere through $O(0,0,0)$ and making intercepts a, b, c on the axis

2) A plane passes through a fixed point (a,b,c) and intercepts the axes in A, B, C. S.T. the centre of the sphere $OABC$ lies on $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$

3) A sphere of constant radius k passes through the origin and intercepts the axes in A, B, C. P.T. the centroid of ΔABC lies on the sphere $9(x^2+y^2+z^2)=4k^2$.

4) S.T. the two circles $x^2+y^2+z^2-y+2z=0$, $x-y+z=2$, $x^2+y^2+z^2+x-3y+z-5=0$, $2x-y+4z-1=0$ lies on the same sphere & find its equ.

5) Find the equ. of the sphere for which the circle $x^2+y^2+z^2+7y-2z+2=0$, $2x+3y+4z=8$ is a great circle.

6) S.T. the plane $2x-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ & find the point of contact.

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TEACHIG NOTES
2019-2020

Name of the Department / Subject : MATHEMATICS

Name of the Lecturer : S. MANIKANTA

urse/Group:	III B.S.C
per:	VI
me of the Topic	Linear transformation
urs required	10
rning Objectives	Concepts of Linear transformation, Properties Range Space, Null Space, kernel etc.
vious knowledge to be minded	Properties of Functions
pic Synopsis	Vectorspace Homomorphism Linear transformation Zero transformation **(Continue on the reverse side if needed)** Negative Transformation Properties of L.T, sum of L.T, Scalar multiplication of L.T, Range space null space, Dimension of Range and kernel
amples/Illustrations	Given on the Reverse side
ditional inputs	
aching Aids used	Blackboard
ferences cited	Telugu Academy and S.Chand
dent Activity planned after the ching	Assignments and Seminars.
tivity planned outside the Class m, if any	
y other activity	

Signature of the Lecturer

Vector space Homomorphism

Let V and U be two vector space over the same field F. Thus the mapping $f: U \to V$ is called a homomorphism from U into V if

(i) $f(\alpha+\beta) = f(\alpha)+f(\beta) \ \forall \alpha, \beta \in U$.

(ii) $f(a\alpha) = af(\alpha) \ \forall a \in F, \alpha \in U$, if f is onto function.

Then V is called homomorphism image of f.

If f is one-one, onto funtion then f is called isomorphism. Thus it is said that U is isomorphic to V denoted by $U \cong V$.

Linear Transformation:-

Let U(F) and V(F) be two vector space. Then the function $T: U \to V$ is called a linear transformations of U into V if $T(a\alpha+b\beta) = aT(\alpha)+bT(\beta) \ \forall a, b \in F, \alpha \beta \in U$.

Zero Transformation:-

Let U(F) and V(F) be two vector spaces let the mapping $T: U \to V$ be defined by $T(\alpha) = \hat{0} \ \forall \alpha \in V$ where $\hat{0}$ (zero raos) is the zero vector of V. Then T is a linear Transformation.

Negative of Transformation:-

Let U(F) & V(F) be two vector space and $T: U \to V$ be a linear Transformation. Then the mapping (-T) defined by $(-T)(\alpha) = -T(\alpha) \ \forall \alpha \in V$ is a linear Transformation.

Properties of Linear Transformation:-

Let $T: U \to V$ is a linear Transformation from the Vectorspace $V(F)$ to the Vectorspace $V(F)$ then

(i) $T(\bar{0}) = \hat{0}$ Where $\hat{0} \in U$ and $\hat{0} \in V$.

(ii) $T(-\alpha) = -T(\alpha)\ \forall\ \gamma \in W$ (iii) $T(\alpha - \beta) = T(\alpha) - T(\beta)\ \forall \alpha \beta \in U$.

(iv) $T(a_1\alpha_1 + a_2\alpha_2 + \dots\dots + a_n\alpha_n) = a_1 T(\alpha_1) + a_2 T(\alpha_2) + \dots + a_n T(\alpha_n)$
$\forall\ a \in F$

Sum of linear Transformations:-

Let T_1 and T_2 be two linear Transformations from U to V. Then their sum $T_1 + T_2$ is denoted by

$$(T_1 + T_2)\alpha = T_1(\alpha) + T_2(\alpha)\ \forall\ \alpha \in V.$$

Scalar Multiplication of a linear transformation

Let $T: U \to V$ be a linear Transformation and $a \in F$. Then the function a^T denoted by

$(a^T)\alpha = aT(\alpha)\ \forall\ \alpha \in V$ is a linear transformation.

Range Space:-

Let $U(F)$ & $V(F)$ be two Vectorspace and Let $T: U \to V$ be a linear Transformation. The range of T is defined to the set $\text{Range}(T) = R(T) = \{T(\alpha) : \alpha \in V\}$.

Null Space (or) Kernel:-

Let $U(F)$ & $V(F)$ be two vector spaces and $T: U \to V$ be a linear transformation. The null space denoted by $N(T)$ is the set of all vectors $\alpha \in V \ni T(\alpha) = \bar{0}\ /\ \bar{0} \in V$.

Dimensions of Range and Kernel:-

Let $T: U(F) \to V(F)$ be a linear Transformation where U is finite dimensional vector space.

Rank:-

Then the rank of T denoted by $\rho(T)$ is the dimension of rangespace $R(T)$ i.e.. $\rho(T) = \dim R(T)$

Nullity:-

The nullity of T denoted by $\nu(T)$ is the dimension of null space $N(T)$. $\nu(T) = \dim N(T)$.

Question Bank

1) Let $U(F)$ & $V(F)$ be two V.S let $T: U \to V$ be a linear Transformation. Then the set $R(T)$ is a Subspace of $V(F)$.
2) Let $U(F)$ & $V(F)$ be two V.S and $T: U \to V$ is a linear Transformation. The null space $N(T)$ is a Subspace of $U(F)$.
3) Let $U: T \to V$ be a linear Transformation. If V is finite dimensional then the range space (T) is finite dimensional Subspace of $V(F)$.
4) Let $U(F)$ & $V(F)$ be two Vectorspace & $T: U \to V$ be a linear Transformation. Let U be the pointed dimensions then $\rho(T) + \nu(T) = \dim U$.

   i.e., $\text{Rank}(T) + \text{Nullity}(T) = \dim U$.

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TEACHIG NOTES
2019-2020

Name of the Department / Subject : MATHEMATICS

Name of the Lecturer : CH·TANUJA

Course/Group: II B.Sc	
Paper: Paper-II	
Name of the Topic	Right line
Hours required	18
Learning Objectives	Find the equation of lines, coplanarity of lines, SD between the 2 skew lines
Previous knowledge to be reminded	Properties of the planes.
Topic Synopsis	equation of straight line, symmetric form of equations of a line, Angle between a line and a plane, Find *(Continue on the reverse side if needed)* the equation of a line (for some conditions) skew lines, SD between skew lines, length of the perpendicular from a point to a line.
Examples/Illustrations	Given on the reverse side
Additional inputs	ICT
Teaching Aids used	Black Board
References cited	S-chand
Student Activity planned after the teaching	Assignments, seminars
Activity planned outside the Class room, if any	
Any other activity	

Ch. Tanuja
Signature of the Lecturer

equation of a straight line is $a_1x+b_1y+c_1z+d_1=0$, $a_2x+b_2y+c_2z+d_2=0$

In symmetric form is $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$.

* The equation of a straight line passing through the point $A(x_1,y_1,z_1)$ and having direction cosines (l,m,n) as $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$.

* The equations of the straight line passing through the point $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ are $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$

* An angle between the line $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ and the plane $ax+by+cz+d=0$ is $\sin^{-1}\left[\frac{|al+bm+cn|}{\sqrt{a^2+b^2+c^2}\sqrt{l^2+m^2+n^2}}\right]$

* The necessary and sufficient conditions for the line $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ to the line in the plane $ax+by+cz+d=0$ are $al+bm+cn=0$ and $ax_1+by_1+cz_1+d=0$

* The necessary and sufficient condition for the lines $\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$ to be coplanar is

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$$

Skew lines :-

Any 2 lines neither intersect nor parallel lines are called skew lines.

Shortest Distance :- let L_1, L_2 be 2 skew lines we know that $\exists$ one and only one line L intersecting $L_1, L_2 \ni L \perp L_1$ & $L \perp L_2$. let L intersect L_1 and M and L_2 at N so that MN is the line segment on L and is between L_1, L_2. Also MN is the shortest Distance between L_1, L_2.

∴ SD between $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ & $\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$

is $$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} \div \sqrt{\Sigma(m_1n_2-m_2n_1)^2}$$

Equations of SD are

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l & m & n \end{vmatrix} = 0 = \begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ l_2 & m_2 & n_2 \\ l & m & n \end{vmatrix}$$

Important questions

1. A variable plane makes intercepts on the coordinate axes the sum of whose squares is constant and is equal to k^2. Find the locus of the foot of the perpendicular from the origin to the plane.

2. Find the equation of the plane through the point $(1,0,-1)$, $(3,2,2)$ and parallel to the line $x-1 = \frac{-y}{2} = \frac{z-2}{3}$.

3. Find the SD and equations of the line of SD b/w the line $\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{3}$ and $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$

4. Find the SD and the equations of the line of SD between the lines $2x+y-z=0, x-y+2z=0$ and $x+2y-3z-4=0$, $2x-3y+4z-5=0$

5. Show that equations to the plane containing the line $\frac{y}{b}+\frac{z}{c}=1$, $x=0$ and parallel to the line $\frac{x}{a}-\frac{z}{c}=1$, $y=0$ is $\frac{x}{a}-\frac{y}{b}-\frac{z}{c}+1=0$ and if '$2d$' is the SD prove that $\frac{1}{d^2} = \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$

6. Find the length and equations of the line of SD between the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{1}$ and $x+y+2z-3=0=2x+3y+3z=4$.

7. Find the length of the $\perp$r from $(4,-5,3)$ to the line $\frac{x-5}{3} = \frac{y+2}{-4} = \frac{z-6}{5}$

Course/Group:	III-Bsc
Paper:	V
Name of the Topic	subrings, Ideals, Quotient rings.
Hours required	11.
Learning Objectives	Properties of groups.
Previous knowledge to be reminded	Properties of groups.
Topic Synopsis	Definition of subrings, Ideals, Principal Ideal. ***(Continue on the reverse side if needed)*** Principal ideal ring. Quotient rings or Factor Ring.
Examples/Illustrations	Given on the Reverse side.
Additional inputs	
Teaching Aids used	Black Board.
References cited	Telugu academy (E) S.chand.
Student Activity planned after the teaching	Assignments and Seminars.
Activity planned outside the Class room, if any	
Any other activity	

K.V.D.S. Prasanna.
Signature of the Lecturer

Subring:- let $(R, +, \cdot)$ be a ring and S be a non-empty subset of R. If $(S, +, \cdot)$ is also a ring w.r.to the two operations $+, \cdot$ in R then $(S, +, \cdot)$ is a subring of R.

Definition:-

let $(F, +, \cdot)$ be a field and $(S, +, \cdot)$ be a subring of F. If $(S, +, \cdot)$ is a field. Then we say that S is a subfield of F. If $(S, +, \cdot)$ is an I.D. Then we say that S is a sub domain of F.

Ideals:-

let $(R, +, \cdot)$ be a ring, A non-empty subset U of R is called a two sided ideal (or) ideal if.

(i) $a, b \in U \Rightarrow a - b \in U$, and.

(ii) $a \in U$ (&) $r \in R \Rightarrow ar, ra \in U$.

Definition:-

A non-empty subset U of a Ring R is called a right ideal if (i) $a, b \in U \Rightarrow a - b \in U$ (&)
(ii) $a \in U, r \in R \Rightarrow ar \in U$.

A non-empty subset U of a ring R is called a right ideal if (i) $a, b \in U \Rightarrow a - b \in U$ (&)
(ii) $a \in U, r \in R \Rightarrow ar \in U$.

A non-empty subset U of a ring R is called a left ideal if (i) $a, b \in U \Rightarrow a - b \in U$. (&)
(ii) $a \in U, r \in R \Rightarrow ra \in U$.

Principal Ideal :-

let R be a commutative ring with unity and $a \in R$. The ideal $\{ra / r \in R\}$ of all multiples of 'a' is called the principal ideal generated by 'a' and is denoted by (a) (or) $\langle a \rangle$.

Principal Ideal Ring :-

A commutative Ring 'R' with unity is a Principal Ideal ring if every ideal in R is a Principal Ideal.

Quotient Rings (or) Factor Rings :-

let R be a ring and U be an ideal of R. Then the set $R/U = \{x+U / x \in R\}$ w.r.to induced operations of addition and multiplication of cosets defined by $(a+U)+(b+U) = (a+b)+U$, $(a+U)(b+U) = ab+U$. for $a+U, b+U \in R/U$ is a ring. This Ring $(\frac{R}{U}, +, \cdot)$ is called the Quotient ring (or) Factor ring (or) Residue class ring of R modulo U.

Question Bank.

① Let 'S' be a non-empty subset of a Ring R. Then 'S' is a subring of R if $a-b \in S$ (&) $a, b \in S$.

② The Intersection of two subrings of a ring R is a subring of R.

③ A field has no proper non-trivial ideals (or) The ideals of a field F are only $\{0\}$ and F it self.

④ If R is a commutative Ring and $a \in R$. Then $Ra = \{ra / r \in R\}$ is an ideal of R.

⑤ A commutative ring R with unity element to a field. if R have no proper ideals.

⑥ The intersection of two ideals of Ring R is an ideal of R.

⑦ If U_1 (&) U_2, are two ideals of a ring R. Then $U_1 \cup U_2$ is an ideal of R if $U_1 \subset U_2$ (or) $U_2 \subset U_1$.

⑧ If U_1, U_2 are two ideals of a Ring R. Then $U_1 + U_2 = \{x+y / x \in U_1, y \in U_2\}$ is also an ideal of $r \in R$.

⑨ The ring of integers Z is a principal Ideal ring. (or)

Every Ideal of Z is a principal Ideal.

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TEACHIG NOTES
2019-2020

Name of the Department / Subject : MATHEMATICS

Name of the Lecturer : K.V.D.S. Prasanna.

Course/Group:	III B.Sc
Paper:	VI
Name of the Topic	Basis and Dimension
Hours required	14
Learning Objectives	Learn about Dimension of vector, Basis & Quotient space, Vector space etc.
Previous knowledge to be reminded	Proporties of vector space.
Topic Synopsis	Basis of vector spaces. Basis extension, Basis of extension, Co-ordinates, Dimensions of a vector space. ***(Continue on the reverse side if needed)*** Dimensions of a vector space. Quotient set. Dimension of a Quotient sub space.
Examples/Illustrations	
Additional inputs	Given on the reverse Side.
Teaching Aids used	
References cited	Black Board.
Student Activity planned after the teaching	Telugu Acadamy and S. Chand
Activity planned outside the Class room, if any	Assignments and Seminars.
Any other activity	

K.V.D.S. Prasanna
Signature of the Lecturer

Basis of Vector space :-

A subset 'S' of a vector space V(F) is said to be basis of V if.

i, S is linearly dependent.

ii, The linear span of S is V.

Finite Dimensional space :-

A vector space V(F) is said to be finite dimensional if it has a finite basis (or)

A vector V(F) is said to be finite dimensional if there is a finite subset S in V such that $L(S) = V$.

Basis Extension :-

let V(F) be a finite dimensional vector space and $S = \{\alpha_1 \alpha_2 \dots \alpha_n\}$ a linearly Independent subset of V. Then either S it self a basis of V (or) S. can be extended to form a basis of V.

If V(F) is a finite dimensional vector space then there exists a basis set of V.

This is Basis existence.

Coordinates :-

let $S = \{\alpha_1 \alpha_2 \dots \alpha_n\}$ be basis set of a finite dimensional vector space V(F) let $\beta \in V$ be given by $\beta = a_1\alpha_1 + a_2\alpha_2 + \dots + a_n\alpha_n \; \forall \; a_1 a_2 \dots a_n \in F$.

Then the scalars $(a_1 a_2 \dots a_n)$ are called the co-ordinates.

Dimensions of a Vector space :-

let $V(F)$ be a finite dimensional vector space. The number of elements in any basis of V is called the dimensions of V and is denoted by $\dim V$.

Dimension of a subspace :-

let $V(F)$ be a finite dimensional vector space of dimensions n and W be a subspace of V. Then W is a finite dimensional vector space with $\dim W \leq n$

let W_1 & W_2 be two subspaces with dimensional vector space $V(F)$ then

$$\dim(W_1 + W_2) = \dim(W_1) + \dim(W_2) - \dim(W_1 \cap W_2).$$

Quotient set :-

let W be a subspace of $V(F)$. Then the set of W in V denoted by $V/W = \{W + \alpha / \forall \alpha \in W\}$ is called Quotient set.

Dimension of a Quotient space :-

let W be a subspace of a finite dimensional vector space $V(F)$. Then $\dim V/W = \dim V - \dim W$.

Question Bank.

1. Let W_1 & W_2 be two subspaces of a finite dimensional vector space $V(F)$. Then.
$\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$.

2. Let N be a subspace of vector $V(F)$. Then the set V/W is a vector space over F, for the vector addition and scalar multiplication defined by.
$(W+\alpha) + (W+\beta) = W + (\alpha+\beta)\ \forall\ \alpha, \beta \in V$.

3. Let W be a subspace of a finite dimensional vector space $V(F)$ then $\dim V/W = \dim V - \dim W$.

4. Problems on linearly dependent and linearly Independent Vectors.

5. Problems on Basis.

6. Basis extension Theorem.

7. Basis Extension Theorem.

Course/Group:	III B.Sc
Paper:	VI
Name of the Topic	Inner Product space.
Hours required	8
Learning Objectives	Learn about Inner product space, norm or length of a vector, & inequalities
Previous knowledge to be reminded	Properties of Vectors.
Topic Synopsis	Inner product space. Norm (or) length of a vector. Parallelogram law. *(Continue on the reverse side if needed)* schwartz Inequality and Important Theorems.
Examples/Illustrations	Given on the Reverse Side.
Additional inputs	
Teaching Aids used	Black Board.
References cited	Telugu Acadamy & s.chand.
Student Activity planned after the teaching	Assignments and seminars.
Activity planned outside the Class room, if any	
Any other activity	

K.V.D.S. Prasanna
Signature of the Lecturer

Inner Product space :-

let V be a finite dimensional vector space over the field F. suppose $F: V \times V \to F$ is a function satisfying the following conditions $\forall x, y, z \in V$, $a b \in F$.

i, $F(x,y)$ is a positive real numbers for each $x(\neq 0) \in V$.

ii, $F(x,y) = F(y,x)$.

iii, $F(ax+by ; z) = aF(x,z) + bF(y,z)$.

Then F is called an Inner product on the Vector space V and (V, F) is called an Inner product space over F. 1, let $V = R^n$ (R) be the n- dimensional euclidan space. for $x = (x_1, x_2 \ldots x_n)$; $y = (y_1, y_2 \ldots y_n)$ in V define $\langle x, y \rangle = x_1 y_1 + x_2 y_2 + \cdots + x_n y_n$.

Now we Verify that No we verify that $\langle \ \rangle$, is an inner product of V let $x = (x_1, x_2 \ldots x_n)$

$y = (y_1, y_2 \ldots y_n)$

$z = (z_1 z_2 \ldots z_n)$ $x, y, z \in R^n$ & $a, b \in R$.

Theorem :-

let V be a inner product. Over field F. If x, y, z $u \in V$ and a, b, c, d, E, F Then we have.

i. If $x = 0$ then for any $y \in V$, $\langle x, y \rangle = 0$

ii, $\langle x . x \rangle = 0$ iff $x = 0$.

iii, $\langle ax + by, cz + dv \rangle = ac \langle x, z \rangle + a\bar{d} \langle x, v \rangle + b\bar{c} \langle y, z \rangle + b\bar{d} \cdot \langle y, v \rangle$.

Norm (or) length of a vector :-

let V be an inner product space over the field F. For any $x \in V$ we define $\|x\| = \sqrt{\langle x, x \rangle}$

Theorem :-

let V be an innerproduct space over the field F. For any $x, y \in V$ and $a \in F$. we have.

i) $\|x\| \geq 0$ ii) $\|x\| = 0$ if $x = 0$ iii) $\|ax\|^2 = |a|^2 . \|x\|^2$.

iv) $\|ax\| = |a| . \|x\|$

Parallelogram law :-

If V be an Innerproduct space over the field F and $x, y \in V$ Then $\|x+y\|^2 + \|x-y\|^2 = 2(\|x\|^2 + \|y\|^2)$.

Schwartz Inequality :-

If V is an innerproduct space over the field F and $x, y \in V$ then $|\langle x, y \rangle| \leq \|x\| , \|y\|$.

Theorem :-

let V be an Inner product space over the field F and $x, y \in V$ then $\|x+y\| \leq \|x\| + \|y\|$.

Theorem :-

If α, β are two vectors in an innerproduct space. Then α, β are L.D if f $|\langle \alpha \beta \rangle| = \|\alpha\| . \|\beta\|$.

Question Bank.

1. Parallelogram law.

2. State and prove Schwartz Inequality Theorem.

3. State and prove orthagonality Theorem.

4. Two vectors α, β is an unitary space $V(C)$ are such that $\langle \alpha, \beta \rangle = 0$ iff $\|a\alpha + b\beta\|^2 = |a|^2\|\alpha\|^2 + |b|^2\|\beta\|^2$ $\forall a, b \in F$.

5. If U, V are two vectors in a complex inner product space with standard inner product then P.T $U\langle U, V\rangle = \|U+V\|^2 - \|U-V\|^2 + \|U+V\|^2 - \|U-V\|^2$.