V.S.M. COLLEGE (AUTONOMOUS)
RAMACHANDRAPURAM

(Re - Accredited by NAAC with 'B' Grade)

TEACHING NOTES
2018 - 2019

Name of the Department / Subject : Mathematics

Name of the Lecturer : N.S.V. Kiran kumar

Course/Group: III BSC	
Paper: VI	
Name of the Topic	Basis and Dimension
Hours required	14
Learning Objectives	
Previous knowledge to be reminded	properties of vector space.
Topic Synopsis	Basis of vector space, Basis extension, Basis of extension, co-ordinates, Dimensions of a vector space. ***(Continue on the reverse side if needed)*** Dimensions of a vector space. Quotient set Dimension of a quotient sub space.
Examples/Illustrations	
Additional inputs	Given on the reverse side.
Teaching Aids used	
References cited	Black Board
Student Activity planned after the teaching	Telugu Academy and S.chand
Activity planned outside the Class room, if any	Assigments and Seminars.
Any other activity	

Signature of the Lecturer

Basis of vector space :-

A subset 's' of a vector space V(F) is said to be basis of V if.

i) S is linearly dependent

ii) The linear span of S is V.

Finite Dimensional Space :-

A vector space V(F) is said to be finite dimensional if it has a finite basis (or)

A vector V(F) is said to be finite dimensional if there is a finite subset 's' in V such that $L(S) = V$

Basis extension;

Let V(F) be a finite dimensional vector space and $S = \{\alpha_1 \alpha_2 --- \alpha_n\}$ a linearly independent subset of V. Then either 's' it self a basis of V (or) S can be extended to form a basis of V.

If V(F) is a finite dimensional vector space then there exists a basis set of V.

This is Basis extence.

Coordinates:

let $S = \{\alpha_1, \alpha_2 --- \alpha_n\}$ be a basis set of a finite dimensional vector space V(F). let $\beta \in V$ be given by

$\beta = a_1\alpha_1 + a_2\alpha_2 + --- + a_n\alpha_n \; \forall \; a_1, a_2 ---- a_n \in F$

Then the scalars $(a_1, a_2 --- a_n)$ are called the co-ordinates.

Dimensions of a vector space :-

let $V(F)$ be a finite dimensional vector space. The number of elements in any basis of V is called the dimensions of V and is denoted by dim V.

Dimension of a subspace :-

let $V(F)$ be a finite dimensional vector space of dimensions n and W be a subspace of V. Then 'W' is a finite dimensional vector space with $\dim W \leq n$.

let W_1 & W_2 be two subspaces with dim 2 dimensional vector space $V(F)$ then.

$$\dim(W_1+W_2) = \dim(W_1) + \dim W_2 - \dim(W_1 \cap W_2)$$

Quotient Set :-

let W be a subspace of $V(F)$. Then the set of W in V denoted by $V/W = \{ W+\alpha \mid \forall \alpha \in V \}$ is called quotient set.

Dimension of a Quotient Space :-

let W be a subspace of a finite dimensional vector space $V(F)$. Then $\dim V/W = \dim V - \dim W$

Question Bank

1. Let W_1 and W_2 be two subspaces of a finite-dimensional vector space $V(F)$. Then

   $\dim(W_1+W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$

2. Let W be a subspace of vector $V(F)$. Then the set V/W is a vector space over F, for the vector addition and scalar multiplication defined by

   $(W+\alpha)+(W+\beta) = W+(\alpha+\beta) \ \forall\ \alpha, \beta \in V$.

3. Let W be a subspace of a finite dimensional vector space $V(F)$. Then $\dim V/W = \dim V - \dim W$

4. Problems on linearly dependent and linearly independent vectors.

5. Problems on Basis.

6. Basis existence theorem.

7. Basis extension theorem.

Course/Group: III B.S.C	
Paper: VII	
Name of the Topic	Interpolation - I
Hours required	10
Learning Objectives	
Previous knowledge to be reminded	Differences Tables.
Topic Synopsis	Forward differences Backward differences central differences *(Continue on the reverse side if needed)* Symbolic Relations and Separation of Symbols
Examples/Illustrations	Given on the reverse side.
Additional inputs	
Teaching Aids used	Black Board
References cited	Telugu Academy & S.Chand
Student Activity planned after the teaching	Assignments and Seminar
Activity planned outside the Class room, if any	
Any other activity	

Signature of the Lecturer

Forward differences:

consider a function $y = f(x)$ of an Independent variable x. let $y_0, y_1, \ldots y_r$ be the value of y corresponding to the values $x_0, x_1, x_2 \ldots x_r$ of x respectively. Then, the difference. $y_1 - y_0, y_2 - y_1, \ldots$ are called the first towards differences of y, and we denoted them by $\Delta y_0, \Delta y_1 \ldots$

that is $\Delta y_0 = y_1 - y_0, \Delta y_1 = y_2 - y_1, \Delta y_2 = y_3 - y_2 \ldots$

In general $\Delta y_r = y_{r+1} - y_r$ - $\because r = 0, 1, 2 \ldots$

Here, the symbol Δ is called the forward difference operator.

The differences of first toward differences are called second toward differences and are denotes by $\Delta^2 y_0, \Delta^2 y_1, \ldots$ that is $\Delta^2 y_0 = \Delta y_1 - \Delta y_0, \Delta^2 y_1 = \Delta y_2 - \Delta y_1 \ldots$

In general $\Delta^2 y_r = \Delta y_{r+1} - \Delta y_r$, $r = 0, 1, 2 \ldots$

Hy, the nth toward differences are defined by the formula

$$\Delta^n y_r = \Delta^{n-1} y_{r+1} - \Delta^{n-1} y_r, \quad r = 0, 1, 2 \ldots$$

Backward differences:-

As mentioned earlier, let $y_0, y_1, y_2 \ldots y_r \ldots$ be the values of a function $y = f(x)$ corresponding to the values $x_0, x_1, x_2 \ldots x_r \ldots$ of x respectively, Then

$\nabla y_1 = y_1 - y_0$, $\nabla y_2 = y_2 - y_1$ --- are called the first backward differences

In general, $\nabla y_r = y_r - y_{r-1}$, $r = 1, 2, 3$ ---

The symbol ∇ is called the backward difference operator. Like the operator Δ, this operator is also a linear operator. It is also called 'nabla'.

The differences of the first backward differences are called second backward differences and are denoted by $\nabla^2 y_2, \nabla^3 y_3$ --- ∇y_r^2 --- i.e,

$$\nabla^2 y_2 = \nabla y_2 - \nabla y_1, \quad \nabla^2 y_3 = \nabla y_3 - \nabla y_2 \;---$$

In general $\nabla^2 y_r = \nabla y_r - \nabla y_{r-1}$; $r = 2, 3$ ---

Ily the nth backward differences are defined by the formula. $\nabla^n y_r = \nabla^{n-1} y_r - \nabla^{n-1} y_{r-1}$

Ily the nth backward differences are defined by the formula

$$\nabla^n y_r = \nabla^{n-1} y_r - \nabla^{n-1} y_{r-1} \; ; \; r = n, n+1 \; ----$$

The symbol ∇^n is referred to as the nth backward difference operator

Question Bank:-

1. construct a Forward difference table from the Following data

x	0	1	2	3	4
y_x	1	1.5	2.2	3.1	4.6

2. Evaluate i, $\Delta \cos x$, ii, $\Delta \log f(x)$, iii, $\Delta^2 \sin(px+q)$ iv, $\Delta \tan^{-1}x$ and v, $\Delta^n e^{ax+b}$

3. prove that $1+\mu^2\delta^2 = \left(1+\frac{1}{2}\delta^2\right)^2$

4. using the method of Separation of Symbols show that

$$\Delta^n u_{x-n} = u_x - n u_{x-1} + \frac{n(n-1)}{2} u_{x-2} + \cdots + (-1)^n u_{x-n}$$

5. The following table gives a set of values of x and the corresponding values of $y=f(x)$

x	10	15	20	25	30	35
y	19.97	21.51	22.47	23.52	24.65	25.89

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TEACHING NOTES
2018 - 2019

Name of the Department / Subject : Mathematics

Name of the Lecturer : N.L.N. MALLIKA.

Course/Group: I B.Sc	
Paper: Paper-II	
Name of the Topic	orthogonal trajectories & with polar Co-ordinates, DE's of first order but not first degree.
Hours required	
Learning Objectives	Solve orthogonal trajectories & O.T of D.E's with Polar Co-ordinates. Solve the first order but not first degree diff - Eqn.
Previous knowledge to be reminded	Properties of differentiations & Integrations.
Topic Synopsis	orthogonal trajectories, Self orthogonal trajectories, Polar co-ordinates. Form of D.E's of first degree and higher degree and Solvable ***(Continue on the reverse side if needed)*** for p, x, y and problems, Clairaut's Equations.
Examples/Illustrations	Given on the back side.
Additional inputs	I.CT
Teaching Aids used	Black board
References cited	S. Chand (Dr. M.D. Raisinghania)
Student Activity planned after the teaching	Seminars and Assignments.
Activity planned outside the Class room, if any	
Any other activity	

Signature of the Lecturer

Trajectory :- If a curve 'c' cuts every member of a given family of curves 'T' according to some specified law, then the curve 'c' is called 'trajectory' of the given family of curves T.

orthogonal trajectories :-

If a curve 'c' cuts every member of a given family of curves T at a right angle is called a orthogonal trajectories of family T.

Polar co-ordinates :-

If $f(r, \theta, c) = 0$, c is being a parameter, is the polar Eq. of the family of curves then the D.E of the family of the orthogonal trajectories is $F(r, \theta, -r^2 \frac{d\theta}{dr}) = 0$

First order D.E. but not first degree :-

An Eq. of the form $\left(\frac{dy}{dx}\right)^n + P_1\left(\frac{dy}{dx}\right)^{n-1} + \dots + P_{n-1}\left(\frac{dy}{dx}\right) + P_n = 0$ is called first order n^{th} degree Diff. Eqs. where $P_1, P_2, \dots P_n$ $\rightarrow$ ① are functions of 'x' and 'y' where $P = \frac{dy}{dx}$ in Eq ① can be written as $P^n + P_1 P^{n-1} + P_2 P^{n-2} + \dots + P_{n-1} P + P_n = 0$.

Eq. ① can also be written as $f(x, y, P) = 0$.

Methode of finding a general solution of $F(x, y, P) = 0$

Method - I :- Solvable for p in terms of x and y :-

Let $f(x, y, P) = 0$ be the n^{th} degree polynomial then $F(x, y, P) = [P - f_1(x, y)][P - f_2(x, y)] \dots [P - f_n(x, y)]$

If $\phi(x, y, c_i) = 0$ is a solution of $P - f_i(x, y) = 0$ for

$i = 1, 2, \dots n$ then $\phi(x, y, c_1), \phi(x, y, c_2) \dots \phi(x, y, c_n) = 0$ is the solution of $F(x, y, p) = 0$.

Method 2 :-

Solvable for 'x'.

If Given D.E $f(x, y, p) = 0$ is of first degree in 'x' then it is solvable for 'x' and it can be written as a function of y and p.

i.e $x = F(y, p) \rightarrow (1)$

Diff. w.r.t. 'y'

$\Rightarrow \frac{dx}{dy} = g(y, p, \frac{dp}{dy})$ which is a first order D.E in 'y' and 'p'.

$\Rightarrow$ Solution of above D.E is $\phi(y, p, c) = 0. \rightarrow (2)$

Eliminating 'p' from (1) & (2) we get a relation between x, y and c which is the required general solution.

Method 3 :- Solvable for 'y' :-

Suppose $f(x, y, p) = 0$ is of first degree in 'y' then it is solvable for 'y' and it can be expressed as a function of x and p. i.e $y = F(x, p) \rightarrow (1)$

diff. w.r.t. 'x'

$\Rightarrow \frac{dy}{dx} = g(x, p, \frac{dp}{dx})$ which is first order D.E in x & p

$\Rightarrow$ Solution for this Equation is $\phi(x, p, c) = 0. \rightarrow (2)$

Eliminating 'p' from Eq. (1) & (2) we get a relation between x, y and c which is required general solution.

Clairauts Equation :-

The D.E of the form $y = xp + f(p)$ is called Clairauts Equation and general solution of Clairauts Eq is $y = xc + f(c)$.

Important Questions

1. Find the orthogonal trajectories of the family of curves $y = \frac{1}{\log c_1 x}$ where c_1 is parameter.

2. Find the orthogonal trajectories of the family of curves $x^{2/3} + y^{2/3} = a^{2/3}$ where 'a' is parameter.

3. Find the orthogonal trajectories of the family of co-axial of circles $x^2 + y^2 + 2gx + c = 0$, where 'g' is a parameter.

4. Show that family of confocal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ is self orthogonal, where 'λ' is a parameter.

5. Find the orthogonal trajectories of the family of curves $r = a(1-\cos\theta)$ where 'a' is a parameter.

6. Find the orthogonal trajectories of the family of curves $r = \frac{2a}{1+\cos\theta}$ where 'a' is parameter.

7. Solve $p^2 + 2py\cot x = y^2$

8. Solve $x^2\left(\frac{dy}{dx}\right)^2 - 2xy\frac{dy}{dx} + 2y^2 - x^2 = 0$

9. Solve $x^2\left(\frac{dy}{dx}\right)^2 + xy\frac{dy}{dx} - 6y^2 = 0$.

10. Solve $y^2 \log y = xpy + p^2$

11. Solve $2px = 2\tan y + p^3\cos^2 y$

12. Solve $y + px = p^2x^4$

13. Solve $y = 2px + x^2p^4$.

14. $y = xp^2 + p$.

15. Solve $(y - xp)(p-1) = p$.

16. Solve $(py + x)(px - y) = 2p$.

Course/Group:	I B.Sc
Paper:	Paper-II
Name of the Topic	The plane
Hours required	18
Learning Objectives	Find the Equation of the plane
Previous knowledge to be reminded	The coordinates.
Topic Synopsis	Eq. of a plane through given points, intercept form, length of perpendicular from a given point to a given plane, **(Continue on the reverse side if needed)** bisectors of angles between two planes, Combined Eq. of two planes, orthogonal projection on a plane.
Examples/Illustrations	Given on the Reverse side
Additional inputs	ICT
Teaching Aids used	Black board
References cited	S. Chand
Student Activity planned after the teaching	Seminars, Assignment.
Activity planned outside the Class room, if any	
Any other activity	

N. LN [illegible]

Signature of the Lecturer

Plane :

A plane is a surface such that is any two points are taken on it. The line joining them lies wholly on the surface.

An equation of the form $ax+by+cz+d=0$, $a,b,c,d \in$ $a^2+b^2+c^2 \neq 0$ is called a real first degree equation in x, y & z where a, b, c are dr's of the normal of the plane.

→ Normal form of the plane is $lx+my+nz=p$.

→ Intercept form of the plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

→ The equation of the plane passing through the origin is of the form $ax+by+cz=0$.

→ The equation of any plane passing through the point (x_1, y_1, z_1) is of the form $a(x-x_1)+b(y-y_1)+c(z-z_1)$

→ If $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ are non-collinear then the equation of the plane determined by A, B, C is

$$\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0.$$

→ Perpendicular distance from (x_1, y_1, z_1) to the plane $ax+by+cz+d=0$ is $\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$.

→ The equation of the plane passing through the point (x_1, y_1, z_1) and parallel to the plane $ax+by+cz+d=0$ is $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$.

→ The foot of the perpendicular $L(p,q,r)$ from a given point $P(x_1,y_1,z_1)$ on to the plane $ax+by+cz+d=0$ is given by

$$\frac{p-x_1}{a} = \frac{q-y_1}{b} = \frac{r-z_1}{c} = \frac{-(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}.$$

→ If $L'(p',q',r')$ is the image of $P(x_1,y_1,z_1)$ with respect to the plane $ax+by+cz+d=0$ then

$$\frac{p'-x_1}{a} = \frac{q'-y_1}{b} = \frac{r'-z_1}{c} = \frac{-2(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}.$$

→ Angle between two planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ is θ then

$$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\cdot\sqrt{a_2^2+b_2^2+c_2^2}}.$$

→ The equation of the planes bisecting the angle between the planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$

$$\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}} = \pm\frac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}}$$

Important Questions.

1. Find the equation of the plane passing through the points $(-1, 2, -2)$; $(0, 1, 1)$, $(1, 1, 2)$ does this plane pass through $(-1, 1, 0)$.

2. If the sum of the reciprocals of the intercepts made by a variable plane on the co-ordinate axes is a non-zero constant. Then show that it passes through a fixed point in all its position.

3. Find the equation of the two planes which passes through the points $(0, 4, -3)$ and $(6, -4, 3)$ and which cut off from the axes intercepts whose sum is zero.

4. A variable plane is at a constant distance 'p' from the origin and meets the co-ordinate axes in A, B, C. Show that the locus of the centroid of the tetrahedron OABC is $x^{-2} + y^{-2} + z^{-2} = 16p^{-2}$.

5. Show that the planes $14x - 8y + 13 = 0$ bisects the obtuse angle between the planes $3x + 4y - 5z + 1 = 0$ and $5x + 12y - 13z = 0$.

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TEACHING NOTES
2018 - 2019

Name of the Department / Subject : MATHEMATICS

Name of the Lecturer : CH. TANUJA

Course/Group: III - BSC

Paper: V

Name of the Topic	Ring Theory
Hours required	10
Learning Objectives	
Previous knowledge to be reminded	Properties of Groups
Topic Synopsis	Definition of ring, Boolean ring, zero divisors of ring, cancellation laws, Integral Domain, Field ***(Continue on the reverse side if needed)*** characteristic of a ring, Divisor(or) Factor, Trivial Divisors, unity element
Examples/Illustrations	Given on the Reverse side
Additional inputs	
Teaching Aids used	Black Board
References cited	Teluge Academy and s.chand
Student Activity planned after the teaching	Assignments and seminar
Activity planned outside the Class room, if any	
Any other activity	

Ch. Tanuja

Signature of the Lecturer

Definition of a ring :- Let R be a non-empty set and $+,\cdot$ be 2 binary operations in R. $(R,+,\cdot)$ is said to be a ring if for $a,b,c \in R$

R_1: $a+b = b+a$ $\quad$ R_2: $(a+b)+c = a+(b+c)$

R_3: $\exists\ 0 \in R \ni a+0=a$ for $a \in R$

R_4: $\exists\ -a \in R \ni a+(-a)=0$ for $a \in R$

R_5: $(a\cdot b)\cdot c = a\cdot(b\cdot c)$ and R_6: $a(b+c) = ab+ac$ and $(b+c)a = ba+ca$

(or)

Let R be a non empty set and $+,\cdot$ be 2 binary operations in R. $(R,+,\cdot)$ is said to be a ring if

(i) $(R,+)$ is a commutative group

(2) $(R,\cdot)$ is semigroup and (3) Distributive Laws hold.

Unity element :- In a Ring $(R,+,\cdot)$ if $\exists\ 1 \in R$ such that $a\cdot 1 = 1\cdot a = a$ for every $a \in R$. Then we say that R is a ring with unity element (or) Identity element.

Definition :- In a ring $(R,+,\cdot)$ if $ab=ba$ for $a,b \in R$ Then we say that R is a Commutative ring.

Characteristic of a ring :- The characteristic of a ring R is defined as the least +ve integer $p \ni pa = 0 \;\forall a \in R$

In case such a +ve integer p does not exist then we say that the characteristic of R is zero (or) +ve.

Boolean Ring :-

In a ring R if $a^2 = a \; \forall a \in R$. Then R is called a Boolean ring.

Zero Divisors:-

Two non-zero elements a, b of a ring R are said to be zero divisors if $ab = 0$ where $0 \in R$ is the zero element. In particular 'a' is a left zero divisor and 'b' is the right zero divisor.

Zero Divisor :- $a \neq 0 \in R$ is a zero divisor if $\exists \, b \neq 0 \in R$ $\ni ab = 0$

Cancellation laws in a Ring :-

In a ring R, for $a, b, c \in R$ if $a \neq 0$. $ab = ac \Rightarrow b = c$ and $a \neq 0$, $ba = ca \Rightarrow b = c$. Then we say that cancellation laws hold in R.

Integral Domain :-

A commutative ring D with unity containing no zero divisors is an integral Domain.

Field :-

Let R be a Commutative ring with unity element If every non-zero element of R is invertible under multiplication. Then R is a field.

Skew field :-

Let R be a ring with unity element. If every non-zero element of R is a unit. Then R is a Division ring.

Question Bank

1. If R is a ring and $0, a, b \in R$ then (i) $0a = a0 = 0$
(2) $a(-b) = (-a)b = -(ab)$ (3) $(-a)(-b) = ab$ and (iv) $a(b-c) = ab-ac$

2. If R is a Boolean ring then (i) $a+a=0$ $\forall a \in R$
(i) $a+b=0 \Rightarrow a=b$ (ii) R is commutative under multiplication

3) A ring R has no zero divisors iff and only iff. The cancellation laws hold in R.

4) A field has non-zero divisors

5) Every field is an integral Domain

6) Every finite Integral Domain is field.

7) Prove that the set $Z(i) = \{a+bi / a, b \in Z, i^2 = -1\}$ of Gaussian integers is an Integral Domain w.r.t addition and multiplication of numbers. Is it a field?

8) Prove that $Q(\sqrt{2}) = \{a+b\sqrt{2} / a, b \in Q\}$ is a field w.r.t ordinary addition and multiplication of numbers

9) The characteristic of an Integral Domain is either a prime or zero

10) The set of 2×2 matrices of the form $\begin{bmatrix} x & y \\ -\bar{y} & \bar{x} \end{bmatrix}$ where x, y are complex numbers and $\bar{x}, \bar{y}$ denote the complex conjugate of x, y is a skew field for compositions of matrix addition and multiplication.

Course/Group:	II B.SC
Paper:	Paper - I
Name of the Topic	Groups
Hours required	12
Learning Objectives	
Previous knowledge to be reminded	
Topic Synopsis	Algebraic structure, Semigroup, Identity element, monoid, Cancellation laws, Group, order of a group, commutator of the ordered pair ***(Continue on the reverse side if needed)*** Addition modulo m, Congruences, Multiplication modulo m.
Examples/Illustrations	Given on the reverse side
Additional inputs	ICT, PPT
Teaching Aids used	Black board
References cited	S. chand
Student Activity planned after the teaching	Seminars, Assignment
Activity planned outside the Class room, if any	
Any other activity	

Ch. Tanuja
Signature of the Lecturer

Algebraic structure :- A non empty set G equipped with one or more binary operations is called an algebraic structure or an algebraic system.

Semigroup :- An algebraic structure (S, o) is called a semi group if the binary operation 'o' is associative in S.

eg :- $(N, +)$ is a semigroup for $a, b \in N \Rightarrow a+b \in N$ & $a, b \in N$.

Identity element :- Let S be a non-empty set and 'o' be a binary operation on S.

(1) If $\exists$ an element $e_1 \in S \ni e_1 o a = a$ for $a \in S$ then e_1 is called a left identity of S w.r.t the operation 'o'.

(2) If there exists an element $e_2 \in S \ni a o e_2 = a$ for $a \in S$ then e_2 is called a right identity of S w.r.t the operation 'o'.

(3) If there exist an element $e \in S \ni e$ is both left and a right identity of S w.r.t 'o', then e is called an identity of S.

MONOID :-

A semigroup (S, o) with the identity element w.r.t 'o' is known as a monoid. i.e, (S, o) is a monoid if S is a non-empty set and 'o' is a binary operation on S such that 'o' is associative and $\exists$ an identity element w.r.t 'o'.

Cancellation laws :-

Let 'S' be a non empty set and 'o' is a binary operation on S.

for $a, b \in S$

(i) $aob = aoc \Rightarrow b = c$ (ii) $boa = coa \Rightarrow b = c$

(i) is called left cancellation law

(ii) is called right cancellation law

(i), (ii) are called cancellation laws.

Group :- If G is a non empty set and 'o' is a binary operation defined on G $\ni$ the following 3 laws are satisfied then (G, o) is a group

G_1 : Associative law :- for $a, b, c \in G \Rightarrow (aob)oc = ao(boc)$

G_2 : Identity law :- $\exists\, e \in G \ni aoe = a = eoa$ for every $a \in G$ is called Identity element in G.

G_3 : Inverse law :- for each $a \in G$ $\exists$ an element $b \in G$ $\ni aob = boa = e$, b is called an inverse of a.

Imp Questions

1. In a group G, Inverse of any element is unique
2. S.T set Q_+ of all +ve rational numbers forms an abelian group under the composition defined by 'o' $\ni$ $a o b = \frac{ab}{3}$ for $a, b \in Q_+$
3. P.T the set G of rational (real) numbers other than 1 with operation $\oplus \ni a \oplus b = a + b - ab$ for $a, b \in G$ is an abelian group. Hence S.T $x = 3/2$ is a solution of the equation $4 \oplus 5 \oplus x = 7$
4. In a group $G (\neq \phi)$ for $a, b, x, y \in G$, the equation $ax = b$ and $ya = b$ have unique solution.
5. A finite semigroup $(G, \cdot)$ satisfying the cancellation laws is a group.
6. P.T the set of nth roots of unity under multiplication form a finite group.
7. The set of non-zero residue classes modulo a prime integer p forms an abelian group of order $(p-1)$ w.r.t multiplication of residue classes.
8. The order of every element of a finite group is finite and is less than or equal to the order of the group.

V.S.M. COLLEGE (AUTONOMOUS)
RAMACHANDRAPURAM

(Re - Accredited by NAAC with 'B' Grade)

TEACHING NOTES
2018 - 2019

Name of the Department / Subject : MATHEMATICS

Name of the Lecturer : G.T Swarna Latha

Course/Group: I BSc	
Paper: Paper-1	
Name of the Topic	Differential Equations of First Order & First Degree
Hours required	08
Learning Objectives	Solve the first order, First degree D.E formation of Differential Equations
Previous knowledge to be reminded	variable separable, homogenous Differential Equations, Linear Differential Equations,
Topic Synopsis	Bernouli's Differential equations, Exact Differential equations, Integrating factor. *(Continue on the reverse side if needed)* Reduce to Exact Differential Equations (Methods), change of variables.
Examples/Illustrations	Given on the Reverse side
Additional inputs	ICT, PPT
Teaching Aids used	Black Board
References cited	S. chand
Student Activity planned after the teaching	Seminors, Assignments
Activity planned outside the Class room, if any	
Any other activity	

G.T. [signature]
Signature of the Lecturer

Linear Differential Equations:-

An equation of the form $\frac{dy}{dx} + Py = Q$ is called linear differential equations.

where P, Q are functions in x.

General solution of linear Differential Equations is

$$y(I.F) = \int Q(I.F)\,dx + C, \text{ where } I.F = e^{\int P dx}$$

Bernouli's Differential Equations:-

An equation of the form $\frac{dy}{dx} + Py = Qy^n \rightarrow (1)$ is called Bernouli's Differential equation; where $n \neq 0$ and $n \neq 1$.

Sol:- $y^{-n}\frac{dy}{dx} + py^{1-n} = Q \rightarrow (2)$

Let $y^{1-n} = v \Rightarrow (1-n)y^{-n}\frac{dy}{dx} = \frac{dv}{dx}$

$$\left.\begin{aligned} &\Rightarrow y^{-n}\frac{dy}{dx} = \frac{1}{1-n}\frac{dv}{dx} \end{aligned}\right\} \rightarrow (3)$$

From (2) & (3) $\frac{dv}{dx} + (1-n)Pv = (1-n)Q$.

This is linear Differential equation.

Exact Differential Equation:-

An equation $Mdx + Ndy = 0$ is called Exact if $\exists\, f(x,y) \ni Mdx + Ndy = d[f(x,y)]$

NOTE:-

$Mdx + Ndy = 0$ is Exact iff $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Solution of Exact D.E:-

$$\int^{x} M dx + \int (\text{terms of } N \text{ not containing } x)\, dy = c$$

Integrating factor:-

Method: I

Let $Mdx + Ndy = 0$ be not an exact D.E.

If $Mdx + Ndy = 0$ can be made exact by multipling it with a suitable function $\mu(x,y) \neq 0$ then $\mu(x,y)$ is called an integrating factor of $Mdx + Ndy = 0$.

Method - II:

$Mdx + Ndy = 0$ is not exact and homogenous

then $I.F = \frac{1}{Mx + Ny}$ where $Mx + Ny \neq 0$

Method - III:

$Mdx + Ndy = 0$ is not exact and $M = yf(x,y)$ and $N = xg(x,y)$ then $I.F = \frac{1}{Mx - Ny}$ where $Mx - Ny \neq 0$

Method - IV:-

$Mdx + Ndy = 0$ is not exact and $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = f(x)$ (or) k then $I.f = e^{\int f(x)dx}$ (or) $e^{\int k dx}$.

Method - V:-

$Mdx + Ndy = 0$ is not exact and $\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = g(y)$ (or) k then $I.F = e^{\int g(y)dy}$ (or) $e^{\int k dy}$

Method: VI

$Mdx + Ndy = 0$ is not exact and $Mdx + Ndy = 0$ is of the form $x^a y^b(my\,dx + nx\,dy) + x^c y^d(py\,dx + qx\,dy) = 0$ then $I.f$ is $x^h y^k$.

Important questions

1) Solve $x\frac{dy}{dx} + 2y - x^2\log x = 0$

2) Obtain the equation of the curve satisfying the D.E $(1+x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ and passing through origin.

3) Solve $(1+y^2)dx = (\tan^{-1}y - x)dy$

4) Solve $x\frac{dy}{dx} + y = y^2\log x$

5) Solve $\frac{dy}{dx}(x^3y^3 + xy) = 1$

6) Solve $(1+e^{x/y})dx + e^{x/y}(1-x/y)\,dy = 0$

7) Solve $x^2y\,dx - (x^3+y^3)\,dy = 0$

8) Solve $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$

9) Solve $(x^2y^2 + xy + 1)y\,dx + (x^2y^2 - xy + 1)x\,dy = 0$

10) Solve $\left(y + \frac{y^3}{3} + \frac{x^2}{2}\right)dx + \frac{1}{4}(x + xy^2)\,dy = 0$

11) Solve $(xy^2 - x^2)dx + (3x^2y^2 + x^2y - 2x^3 + y^2)\,dy = 0$

Course/Group: II B.Sc	
Paper: IV	
Name of the Topic	sequences
Hours required	20
Learning Objectives	
Previous knowledge to be reminded	Properties of Real numbers
Topic Synopsis	sequences and their limits, limit theorems, monotonic sequences limits of an sequences, sub-sequences and *(Continue on the reverse side if needed)* the Bolzano-weirstrass thm, the cauchy's criterion, Properly divergent sequences.
Examples/Illustrations	Given on the Reverse side.
Additional inputs	ICT, PPT
Teaching Aids used	Black board.
References cited	S.chand, Real Analysis, S.chand
Student Activity planned after the teaching	Assignment and seminars
Activity planned outside the Class room, if any	
Any other activity	

G.T.[signature]

Signature of the Lecturer

* Limit of a sequence:-

Let $\{S_n\}$ be an sequences and $l \in \mathbb{R}$, 'l' is said to be the limit of the sequence $\{S_n\}$ if to each $\varepsilon > 0$ $\exists$ $m \in \mathbb{Z}^+$ such that $|S_n - l| < \varepsilon$, $\forall$ $n \geq m$. we also say that the seqn $\{S_n\}$ cgs to l.

If the sequences $\{S_n\}$ has the limit

Field axioms:-

The set of real numbers $\mathbb{R}$ is a field with two binary operations called addition (+) and multiplication $(\cdot)$ which satisfy the following axioms $A_1, A_2, A_3, A_4, M_1, M_2, M_3, M_4$ & D

A_1: $a+b = b+a, \forall a, b \in \mathbb{R}$

A_2: $(a+b)+c = a+(b+c), \forall a, b, c \in \mathbb{R}$

A_3: $\mathbb{R}$ contains a unique element '0' $\ni$ $a+0=a$, for every $a \in \mathbb{R}$

A_4: For any $a \in \mathbb{R}$ there corresponds a unique element $-a \in \mathbb{R}$ such that $a+(-a)=0$. "$-a$" is called negative of 'a' or additive inverse

Illy M_1, M_2, M_3, M_4

The order axioms:-

The order relation ">" defined for pairs of real numbers satisfies the following axioms.

O_1: for $a, b \in \mathbb{R}$ one & only one of the following true

$a > b$, $a = b$, $b > a$ (law of Trichotomy)

O_2: for $a, b, c \in \mathbb{R}$, $a > b$, $b > c \Rightarrow a > c$ (Transitivity)

O_3: for $a, b, c \in \mathbb{R}$, $a > b \Rightarrow a+c > b+c$

O_4: for $a, b, c \in \mathbb{R}$, $a > b$, $c > 0 \Rightarrow ac > bc$

Least upper Bound (l.u.b) or supremum:-

If "u" is an upper bound of an aggregate S and any real number less than 'u' is not an upper bound of S, Then u is called least upper bound or supremum of S.

Lower bound:-

An aggregate "S" is said to be bounded below, if $\exists$ $k_2 \in \mathbb{R} \ni x \in S \Rightarrow x \geq k_2$. The number k_2 is called a lower bound of S.

Greatest lower bound (g.l.b) or Infimum:-

If v is a lower bound of an aggregate S and any real number greater than v is not a lower bound of S. Then v is called greatest lower bound or infimum of S.

Boundedness:-

An aggregate 'S' is said to be bounded if it is both bounded below and bounded above.

Subsequences:-

If $\{S_n\}$ is a sequence and $\{n_n\}$ is a sequence of positive integers such that $n_1 < n_2 < n_3 < \cdots$, Then the sequence $\{S_{n_n}\}$ is called subsequence of $\{S_n\}$.

Range of sequence:-

The set of all terms of a sequences is called range set or range of the sequence. Range of set of $\{S_n\} = \{S_n / n \in \mathbb{Z}^+\}$.

Monotone sequence:-

A seq $\{S_n\}$ which is either increasing (or) decreasing is called monotone sequence.

Important Qustions

1) A sequence can have atmost one limit (or) A convergence sequence has unique limit.

2) state and prove sandwich theorem (or) squeeze theorem.

3) A monotone sequences is convergent iff it is bounded.

4) P.T. the seq. $S_n = \frac{3n+4}{2n+1}$ is decreasing and bounded below.

5) P.T. the seq. $\{S_n\}$ where $S_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n}$ is convergent.

6) state and prove Bolzano - weirstrass thm.

7) If the ~~eq~~ seq $\{S_n\}$ is cgt, then $\{S_n\}$ is a cauchy seq.

8) state & prove cauchy's general principle of convergence (or) cauchy's thm.

9) If $\{S_n\}$ is a cauchy seq. then $\{S_n\}$ is cgt.

10) state and prove cauchy's first thm on limits.

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TEACHING NOTES
2018 - 2019

Name of the Department / Subject : Mathematics

Name of the Lecturer : S. Manikanta

Course/Group: III Bsc	
Paper: VI	
Name of the Topic	Linear Transformation.
Hours required	
Learning Objectives	
Previous knowledge to be reminded	Properties of functions of Subspaces.
Topic Synopsis	Linear Transformation, Linear Operations Properties of L.T, Sum and product of LTS, Algebra of Linear Operators. **(Continue on the reverse side if needed)** Range and Null space of LT. Rank and Nullity of L.Ts. Rank and Nullity Theorem.
Examples/Illustrations	Given on the Reverse side
Additional inputs	
Teaching Aids used	Black board
References cited	S.chand
Student Activity planned after the teaching	Assignment and Seminars.
Activity planned outside the Class room, if any	
Any other activity	

Signature of the Lecturer

Linear Transformation :->

Let $U(F)$ and $V(F)$ be two vector spaces. Then the function $T: U \to V$ is called a Linear transformation of U into V if

$$T(a\alpha + b\beta) = aT(\alpha) + bT(\beta) \ \forall a, b \in F;\ \alpha, \beta \in U.$$

Linear Operator :->

If $T: U \to U$ is L.T then that L.T is called as linear operator.

Sum of L.T's :->.

Let T_1 and T_2 be two LTs from $U(F)$ to $V(F)$. Then their Sum $T_1 + T_2$ is denoted by $(T_1 + T_2)(\alpha) = T_1(\alpha) + T_2(\alpha) \ \forall \alpha \in U$.

Scalar Multiplication of LT :->

Let $T: U \to V$ be LT and $a \in F$. Then the function aT is denoted by

$$(aT)(\alpha) = aT(\alpha) \ \forall \alpha \in U.$$

Product of LTs :->

Let $U(F)$, $V(F)$ & $W(F)$ are 3 Vector spaces & $T: V \to W$ & $H: U \to V$ are LTs Then the product (composition) of T & H is defined by $(TH)(\alpha) = T[H(\alpha)] \ \forall \alpha \in U$

Algebra of Linear Operators

Let A, B, C be Linear Operators on a Vector Space $V(F)$ and O is Zero-Operator, I is Identity operator on V.

Then (i) $AO = OA = O$ (ii) $AI = IA = A$

(iii) $A(B+C) = AB + AC$ (iv) $A(BC) = (AB)C$.

Range :- Let $U(F)$ and $V(F)$ be 2 Vector spaces and let $T: U \to V$ be L.T The range of T is defined to be the set

$$Range(T) = R(T) = \{T(\alpha) / \alpha \in U\}.$$

Null space (or) Kernel :- Let $U(F)$ & $V(F)$ be two vectorspaces and $T: U \to V$ be L.T. The null space denoted by $N(T) \ni N(T) = \{\alpha \in U / T(\alpha) = \bar{0} \in V\}$.

Rank :- Let $T: U \to V$ be L.T & U is F.D.V.S. Then the rank of T is denoted by $\rho(T)$ and dimension of Range Space $R(T)$ is Rank of T i.e.,

$$\rho(T) = \dim R(T).$$

Nullity :- The nullity of T denoted by $\nu(T)$ is the dimension of Null space $N(T)$

i.e., $\nu(T) = \dim N(T)$.

important Questions :-)

1) Let $V(F)$ and $U(F)$ be two V.S. Let $T: U \to V$ be a L.T then S.T $R(T)$ is a Subspace of $V(F)$.

2) Let $U(F)$ and $V(F)$ be two V.S. Let $T: U \to V$ be a L.T then S.T $N(T)$ is a Subspace of $U(F)$.

3) Let $T: U \to V$ be a L.T, $U(F)$ and $V(F)$ be two V.S and F is a field. Let U be a F.D.V.S. Then $\rho(T) + \nu(T) = \dim U$ i.e.,

$$\text{Rank}(T) + \text{nullity}(T) = \dim U.$$

Course/Group: III BSc	
Paper: VII	
Name of the Topic	Solution of Algebric and trancendental Equation.
Hours required	18
Learning Objectives	
Previous knowledge to be reminded	Find out the roots
Topic Synopsis	Bisection Method Regular falsi method Newton-Raphson Method **(Continue on the reverse side if needed)** Generalized - Newton Raphson Method. Iteration - Method
Examples/Illustrations	Given on the reverse side
Additional inputs	
Teaching Aids used	Black board
References cited	Telugu Acadamy, S. Chand and Deepthi
Student Activity planned after the teaching	Assignment & Seminars.
Activity planned outside the Class room, if any	
Any other activity	

Signature of the Lecturer

Bisection Method :->

If $f(x_0)=0$ we find a root α lies b/w (a,b). we consider the midpoint $x_0 = \frac{a+b}{2}$ of $[a,b]$. If $f(x_0)=0$ then the required root is $\alpha = x_0$ otherwise the process is continued till the required accuracy is obtained

Regula - falsi method :->

$$x_{i+1} = \frac{x_{i-1} f(x_i) - x_i f(x_{i-1})}{f(x_i) - f(x_{i-1})}$$

(or)

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}.$$

Newton-Raphson Method :->

General formula of Newton Raphson Method is

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}.$$

Generalized Newton Raphson method :->

Let $\phi(x,y)=0$, $\psi(x,y)=0$ be two functions. the generalized Newton-Raphson method is

$$h\left[\frac{\delta\phi}{\delta x}\right]_n + k\left[\frac{\delta\phi}{\delta y}\right]_n = -\phi_n$$

$$h\left[\frac{\delta\psi}{\delta x}\right]_n + k\left[\frac{\delta\psi}{\delta y}\right]_n = -\psi_n$$

Iteration Method :-

Consider an Eqn $f(x)=0$ which can take in the form $x=\phi(x)$ then to find approximate value of this root, we start with an approximation x_0, then n^{th} approximation is given by $x_n=\phi(x_{n-1})$.

Question Bank :-)

1) Using Bisection Method find the roots of the Eqn $x^3 - x - 4 = 0$ correct to 4 decimal places.

2) Find the root of the Eqn $x^3 - x^2 - 1 = 0$ by falsi position Method.

3) Use Newton Raphson Method to find a root of the Eqn $x^3 - 3x - 5 = 0$ to 3 decimal places

4) Using Generalized Newton's Raphson Method find a real solution of $x^2 - y^2 = 4$; $x^2 + y^2 = 6$ with $[x_0, y_0] = [2.828; 2.828]$ [Up to 2 iteration]

5) Solve the Eqn $\sin x = 5x - 2$ by iteration Method upto 4 decimal places.