V.S.M. COLLEGE, RAMACHANDRAPURAM

(Re Accredited by NAAC with 'B' Grade)

TEACHING NOTES
2017 - 2018

Name of the Department / Subject : MATHEMATICS

Name of the Lecturer : N.S V. Kiran kumar.

Course/Group:	III - BSC.
Paper:	V
Name of the Topic	Ring Theory.
Hours required	10
Learning Objectives	Intigrate functions of several variables over curves and surfaces
Previous knowledge to be reminded	Properties of groups.
Topic Synopsis	Definition of ring, Boolean ring, zero Divisors of ring, Cancellation laws Integral domain. field, *(Continue on the reverse side if needed)* characteristic of a ring, Divisor (or) Factor, Trivial Divisors, unity element.
Examples/Illustrations	Given on the Reverse side.
Additional inputs	
Teaching Aids used	Black Board.
References cited	Telugu Academy and S.chand
Student Activity planned after the teaching	Assignments and seminar
Activity planned outside the Class room, if any	
Any other activity	

Signature of the Lecturer

Definition of Ring:

Let R be a non-empty set and $+, \cdot$ be two binary operations in R. $(R, +, \cdot)$ is said to be a ring if for $a, b, c \in R$.

R_1 : $a+b = b+a$. R_2 : $(a+b)+c = a+(b+c)$

R_3 : $\exists\, 0 \in R \ni a+0 = a$ for $a \in R$.

R_4 : $\exists\, -a \in R \ni a+(-a) = 0$ for $a \in R$.

R_5 : $(a \cdot b) \cdot c = a \cdot (b c)$ and R_6 : $a(b+c) = ab + bc$ and
$(b+c)a = ba + ca$.

(or)

Let R be a non-empty set and $+, \cdot$ be two binary operations in R. $(R, +, \cdot)$ is said to be a ring if i, $(R, +)$ is a commutative group.
(ii, $(R, \cdot)$ is semigroup. and
(iii, Distributive laws hold.

Unity Element:

In a ring $(R, +, \cdot)$ if $\exists\, 1 \in R$ such that $a \cdot 1 = 1 \cdot a = a$ for every $a \in R$. Then we say that R is a ring with unity element or Identity element.

Definition:

In a ring $(R, +, \cdot)$ if $ab = ba$ for $a, b \in R$ Then we say that R is commutative ring.

Boolean Ring:

In a ring R if $a^2 = a \ \forall a \in R$ Then R is Called a Boolean ring.

Zero Divisors:

Two non-zero elements a, b of a ring R are said to be zero divisors if $ab = 0$ where $0 \in R$ is the zero element

In Particular 'a' is left zero divisor and 'b' is right zero divisor.

Zero Divisors:

$a \neq 0 \in R$ is a zero divisor if $\exists\, b \neq 0 \in R \ni ab = 0$

Cancellation laws in a Ring:

In a ring R, for $a, b, c \in R$ if $a \neq 0$.

$ab = ac \Rightarrow b = c$. and $a \neq 0$, $ba = ca \Rightarrow b = c$ Then we say that Cancellation laws hold in R.

Integral Domain:

A Commutative ring D with unity containing no zero divisors is an Integral domain.

Field:

Let R be a Commutative ring with unity element If every non-zero element of R is invertible under multiplication Then R is a field.

Skew field:

Let R be a ring with unity element. If every non-zero element of R is a unit Then R is a Division ring.

Characteristic of ring:

The characteristic of a ring R is defined as the least +ve Integer $p \ni pa = 0 \ \forall a \in R$. In case such a positive Integer p does not exist Then we say that the Ch. of R is zero (or) infinite.

Question Bank.

1. If R is a ring and $0, a, b \in R$ Then i, $0a = a0 = 0$.
ii, $a(-b) = (-a)b = -(ab)$ iii, $(-a)(-b) = ab$ and iv, $a(b-c) = ab - ac$.

2. If R is a Boolean ring Then i, $a + a = 0 \ \forall a \in R$.
ii, $a + b = 0 \Rightarrow a = b$ iii, R is Commutative under multiplication.

3. A ring R has no zero divisors iff and only iff The Cancellation laws hold in R.

4. A field has no zero - divisors.

5. Every field is an Integral domain.

6. Every finite Integral domain is field.

7. prove that the set $Z(i) = \{a + bi / a, b \in Z, i^2 = -1\}$ of Gaussian Integers in an integral domain w.r.t. addition and multiplication of numbers Is it a field?

8. prove that $Q(\sqrt{2}) = \{a + b\sqrt{2} / a, b \in Q\}$ is a field w.r.t ordinary addition and multiplication of numbers.

9. The characteristic of an Integral domain is either a prime or zero.

10. The set of 2×2 matrices of the form $\begin{bmatrix} x & y \\ -\bar{y} & \bar{x} \end{bmatrix}$ where x, y are Complex numbers and $\bar{x}, \bar{y}$ denote the Complex Conjugate of x, y is a skew field for Compositions of matrix addition and multiplication.

V.S.M. COLLEGE (AUTONOMOUS)
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TEACHING NOTES
2017 - 2018

Name of the Department / Subject : Mathematics

Name of the Lecturer : N. L. N. Mallika

Course/Group: II B.sc	
Paper: Paper - III	
Name of the Topic	Subgroups, Cosets and Lagrange's theorem
Hours required	08.
Learning Objectives	Analyze and demonstrate examples of subgroups, normal subgroups and quotient groups
Previous knowledge to be reminded	properties of groups.
Topic Synopsis	Complex definition, Subgroups, Criterion for the product of two subgroups to be a subgroup, union and intersection of ***(Continue on the reverse side if needed)*** subgroups, Definition of coset, Properties of cosets, Normalizer of an element of a group.
Examples/Illustrations	Given on the reverse side.
Additional inputs	ICT
Teaching Aids used	Black board.
References cited	S. chand.
Student Activity planned after the teaching	Seminars, Assignment.
Activity planned outside the Class room, if any	
Any other activity	

N. L. N. [signature]

Signature of the Lecturer

Complex definition:

Any subset of a group G is called a complex of G.

Eg: The set of integers is a complex of the group $(R,+)$

Multiplication of two complexes :-

If M and N are any two complexes of a group G then $MN = \{mn \in G \mid m \in M, n \in N\}$

clearly, $MN \subseteq G$ and MN is called the product of the complexes M, N of G.

Subgroups :-

Let $(G, \cdot)$ be a group. Let H be a non-empty subset of G such that $(H, \cdot)$ be a group. Then H is called a subgroup of G.

It is denoted by $H \leq G$ (or) $G \geq H$ and $H < G$ or $G > H$ we mean $H \leq G$ but $H \neq G$.

Union and Intersection of subgroups :-

If H_1 and H_2 are two subgroups of a group G then $H_1 \cap H_2$ is also a subgroup of G.

Cosets :-

Let $(H, \cdot)$ be a subgroup of the group $(G, \cdot)$. Let $a \in G$ then the set $aH = \{ah \mid h \in H\}$ is called a left coset of H in G generated by 'a' and the set

Complex definition :

Any subset of a group G is called a Complex of G.

Eg: The set of integers is a complex of the group $(R,+)$

Multiplication of two Complexes :-

If M and N are any two complexes of a group G then $MN = \{mn \in G \mid m \in M, n \in N\}$

clearly, $MN \subseteq G$ and MN is called the product of the Complexes M, N of G.

Subgroups :

Let $(G,\cdot)$ be a group. Let H be a non-empty subset of G such that $(H,\cdot)$ be a group. Then H is called a sub group of G.

It is denoted by $H \leq G$ (or) $G \geq H$ and $H < G$ or $G > H$ we mean $H \leq G$ but $H \neq G$.

union and Intersection of subgroups :-

If H_1 and H_2 are two subgroups of a group G then $H_1 \cap H_2$ is also a sub group of G.

Cosets :

Let $(H,\cdot)$ be a subgroup of the group $(G,\cdot)$ Let $a \in G$ then the set $aH = \{ah \mid h \in H\}$ is called a left coset of H in G generated by 'a' and the set

$Ha = \{ha / h \in H\}$ is called a right coset of H in G generated by 'a'. Also aH, Ha are called cosets of H generated by 'a' in G.

Congruence modulo H :-

Let $(G, \cdot)$ be a group and $(H, \cdot)$ be a subgroup of G. For $a, b \in G$ if $b^{-1}a \in H$ we say that $a \equiv b \pmod H$.

Normaliser of an element of a group :-

If 'a' is an element of a group G, then the Normalizer of 'a' in G is the set of all those elements of G which commute with 'a'.

The normalizer of 'a' in G is denoted by N(a) where $N(a) = \{x \in G / ax = xa\}$

The normalizer N(a) is a subgroup of G.

Self-conjugate element of a group :-

$(G, \cdot)$ is a group and $a \in G$ such that $a = x^{-1}ax, \forall x \in G$. Then 'a' is called self conjugate element of G. A self-conjugate element is sometimes called an invariant element.

Imp Questions

1. If H is any subgroup of a group G, then $H^{-1} = H$.
2. H is a non-empty complex of a group G. The necessary and sufficient condition for H to be a subgroup of G is $a, b \in H \Rightarrow ab^{-1} \in H$ where b^{-1} is the inverse of 'b' in G.
3. The necessary and sufficient condition for a finite complex H of a group G to be a subgroup of G is $a, b \in H \Rightarrow ab \in H$.
4. If H and K are two subgroups of a group G, then HK is a subgroup of G iff $HK = KH$.
5. If H_1 & H_2 are two subgroups of a group G then $H_1 \cap H_2$ is also a subgroup of G.
6. The union of two subgroups of a group is a subgroup iff one is contained in the other.
7. If a, b are any two elements of a group $(G, \cdot)$ and H any subgroup of G then $Ha = Hb \Leftrightarrow ab^{-1} \in H$ and $aH = bH \Leftrightarrow a^{-1}b \in H$.
8. Any two left (right) cosets of a subgroup are either disjoint or identical.
9. State and prove Lagrange's theorem.

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TEACHING NOTES
2017 - 2018

Name of the Department / Subject : MATHEMATICS

Name of the Lecturer : V. L. Sudhakar

Course/Group: I B.SC	
Paper: Paper-I	
Name of the Topic	Linear Diff^{al} Eq^{n} with Variable coefficients
Hours required	
Learning Objectives	Solve $f(D)y=0$
Previous knowledge to be reminded	Find $f(D)y=0$ solution and particular integration of $f(D)y=Q(x)$
Topic Synopsis	Method of Variation of parameters Solve Linear Diff^{al} eq^{ns} with non-constant coefficients by *(Continue on the reverse side if needed)* Method of Reduction of order method The Cauchy-Euler's eq^{n} & Legender's Eq^{ns}
Examples/Illustrations	Given on the Reverse Side
Additional inputs	ICT
Teaching Aids used	Black Board
References cited	S.chand (Dr. M.D. Raisingana)
Student Activity planned after the teaching	Seminars, Assignments.
Activity planned outside the Class room, if any	
Any other activity	

Signature of the Lecturer

Method of Variation of parameters:-

Second order Linear Diff^al Eqn is

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)\,y = R(x) \rightarrow \boxed{1}$$

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)\,y = 0 \rightarrow \boxed{2}$$

The C.F $= y_c = c_1 u(x) + c_2 v(x)$.

Suppose $y_p = A\,u(x) + B\,v(x)$

where $A = -\int \frac{vR}{W}$ $\quad B = \int \frac{uR}{W}$

where $W = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix} = uv' - u'v$.

The G.sol eqn $\boxed{1}$ is $y = c_1 u(x) + c_2 v(x) + A\,u(x) + B\,v(x)$

To Solve Linear Diff^al Eqn with non-Constant Coefficient by the Method of Reduction.

The Cauchy-Euler Eqn:-

An Eqn of the form

$$a_n x^n \frac{d^ny}{dx^n} + a_{n-1} x^{n-1} \frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_1 x \frac{dy}{dx} + a_0 y = Q(x)$$

is called a Cauchy-Euler Eqn of order n

where $a_0, a_1, \ldots a_n$ are constants

Second order eqn is $a_2 x^2 \frac{d^2y}{dx^2} + a_1 x \frac{dy}{dx} + a_0 y = Q(x)$ $\quad 2\,\boxed{1}$

put $x = e^t \Rightarrow \log x = t$

Then $x\dfrac{dy}{dx} = \dfrac{dy}{dt}$ or D $\Bigg\} \to \boxed{2}$

$$x^2\frac{d^2y}{dx^2} = \frac{d^2y}{dt^2} - \frac{dy}{dt} \text{ (or) } x^2D = D(D-1)$$

From $\boxed{1}$ & $\boxed{2}$

$$a_2\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) + a_1\left(\frac{dy}{dt}\right) + a_0 y = Q(e^t)$$

$$\Rightarrow a_2\frac{d^2y}{dt^2} + (-a_2+a_1)\frac{dy}{dt} + a_0 y = Q(e^t)$$

$$A_2\frac{d^2y}{dt^2} + A_1\frac{dy}{dt} + A_0 y = Q(e^t) \longrightarrow \boxed{3}$$

where $A_2 = a_2$; $A_1 = a_1 - a_2$; $A_0 = a_0$

clearly Eqn $\boxed{3}$ in Linear Diff^{al} Eqn with constant coefficients. It can be solve Short cut Methods of $f(D)y = Q(x)$.

<u>Note</u> :- $x^3D^3 = D(D-1)(D-2)$

The Legendern Eqn :-

An Eqn of the form

$$(ax+b)^n\frac{d^ny}{dx^n} + (ax+b)^{n-1}\frac{d^{n-1}y}{dx^{n}} + \cdots + (ax+b)^1\frac{dy}{dx} + y = Q(x)$$

Let $ax+b = e^z \Rightarrow z = \log(ax+b)$

Then

$$(ax+b)\frac{dy}{dx} = a\cdot D$$

$$(ax+b)^2\frac{dy}{dx} = a^2\cdot D(D-1)$$

$$(ax+b)^3\frac{dy}{dx} = a^3D(D-1)(D-2)$$

$$\vdots$$

$$(ax+b)^n\frac{d^ny}{dx^n} = a^nD(D-1)(D-2)\cdots(D-(n-1))$$

Important Questions

① Find the particular integral of $y'' + 4y = 2\tan x$ by using the method of variation of parameters

② $(D^2 - 2D)y = e^x \sin x$ by the Method of Variation of parameters

③ Given $y_1(x) = x$ as a soln $x^2y'' + xy' - y = 0$. find the general soln of $x^2y'' + xy' - y = x$.

④ Solve $x^2y'' + 3xy' + y = 0$ G.T $y_1 = \frac{1}{x}$ is a soln

⑤ Solve $x^2y''' = 2y'$

⑥ Solve $x^2y'' + 7xy' - 7y = x^2 + x$

⑦ $x^2y'' - xy' + y = 2\log x$

⑧ $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 11x^3$.

⑨ $[(3x+2)^2D^2 + 3(3x+2)D - 36]y = 3x^2 + 4x + 1$,

⑩ $(1+x^2)(d^2y/dx^2) + (1+x)(dy/dx) + y = 4\cos\log(1+x)$.

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TEACHING NOTES
2017 - 2018

Name of the Department / Subject : MATHEMATICS

Name of the Lecturer : D.S.V Harika

ourse/Group: I B.Sc	
aper: paper - I	
Name of the Topic	The sphere
Hours required	19
Learning Objectives	Find the Equation of spheres and properties of spheres
Previous knowledge to be reminded	properties of the straight line
Topic Synopsis	Definition of sphere, area of the sphere, Intersection of a plane and sphere, great circle, Tangent plane, polar plane, plane of contact, power point, ***(Continue on the reverse side if needed)*** conjugate points, Intersection of two spheres, orthogonal spheres, orthogonal spheres, Radical plane, co-axial system
Examples/Illustrations	Given on the reverse side
Additional inputs	ICT
Teaching Aids used	Black Board
References cited	S. chand
Student Activity planned after the teaching	Assignment, Seminar
Activity planned outside the Class room, if any	
Any other activity	

Signature of the Lecturer

Sphere: The set of points in a space which are at a constant distance $a (\geq 0)$ form a fixed point c is called a sphere

→ Equation to the sphere with center (x_1, y_1, z_1) and radius 'a' is $(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2 = a^2$

→ Equation to sphere is of the form $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ where radius $= \sqrt{u^2+v^2+w^2-d}$ and center c $(-u, -v, -w)$.

concentric sphere:

Sphere with the same center are known as concentric sphere.

→ A plane section of a sphere is a circle

→ Great circle:

If a plane π passes through the center of a sphere S, then the plane section of the sphere is called a great circle.

→ If two spheres intersect, then the locus of the set of points of intersection and a circle

→ length of the tangent line from (x_1, y_1, z_1) to the sphere $S=0$ is $\sqrt{S_{11}}$

→ Equation of the plane of contact of all point (x_1, y_1, z_1) with respect to the Sphere $S=0$ of non-zero radius S is $S_1=0$.

→ The Equation of the polar plane of the point (x_1, y_1, z_1) with respect to the sphere $S=0$ is $S_1=0$.

→ If $x^2+y^2+z^2-a^2=0$ is a sphere, then the pole of the plane $lx+my+nz=p$ is

$$\left(\frac{a^2 l}{p}, \frac{a^2 m}{p}, \frac{a^2 n}{p}\right)$$

Angle of intersection of sphore:

P is common point to 2 spheres S_1, S_2

Any angle 'θ' between the tangent planes at P to two sphore is called an angle of intersection of the sphory. S_1, S_2 at P then other angle between the sphore is $\pi - \theta$.

→ Equation to the radical plane of sphere $S = 0$, $S' = 0$ is $S - S' = 0$.

→ If $S = 0$ is a sphere and $U = 0$ is a plane, then the Equation $S + \lambda U = 0$ represent a co-axial system of sphere with radical plane $U = 0$.

N J Bora

Important questions

① The plane of $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the axes in A, B, C. Find the Equation of the circumcircle of ΔABC and hence Find its center

② Find great circle (or) small circle
$x^2+y^2+z^2-4x+6y-8z+4=0$, $x+y+z=3$

③ Show that the plane $2x-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x+4y+2z-3=0$ and Find the point of contact.

④ Find the pole of the plane $10x-2y-5z-2=0$ with respect to sphere $x^2+y^2+z^2-6x+2y-2z+1=0$.

⑤ S_1, S_2 are two intersecting spheres ; r_1, r_2 are their respective radii and d is the distance between their centers. If p is a common point to S_1, S_2 then an angle θ of intersection of the sphere S_1, S_2 at p is given by

$$\cos\theta = \pm\frac{(r_1^2+r_2^2-d^2)}{2r_1r_2}$$

⑥ If r_1, r_2 are the radii of two orthogonal sphery, then the radius of the circle of their intersection. is $\frac{r_1r_2}{\sqrt{r_1^2+r_2^2}}$

MJ Kranz

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TEACHING NOTES
2017 - 2018

Name of the Department / Subject : MATHEMATICS

Name of the Lecturer : G. N. Uma Maheswari

ourse/Group:	III B.Sc
aper:	VI
ame of the Topic	orthogonality.
ours required	8.
earning Objectives	
revious knowledge to be minded	Properties of inner product space.
opic Synopsis	orthogonal orthogonal set orthonormal. Gram Schmidt orthogonalisation process. ***(Continue on the reverse side if needed)*** orthogonal complement and Important Theorems.
xamples/Illustrations	Given on the reverse side.
dditional inputs	
aching Aids used	Black Board.
eferences cited	Telugu Acadamy & S.chand.
udent Activity planned after the aching	Assignments & Seminars
tivity planned outside the Class om, if any	
y other activity	

G.N.V.
Signature of the Lecturer

orthogonal:

Let $V(F)$ be an inner product space and $\alpha, \beta \in V$ is said to be orthogonal to β if $\langle \alpha, \beta \rangle = 0$

'α' is orthogonal to ~~β~~ $\beta \Leftrightarrow \langle \alpha \beta \rangle = 0 \Leftrightarrow$ $\langle \alpha, \beta \rangle = 0 \Leftrightarrow \beta$ is orthogonal to α.

Theorem:

If the vectors α, β of an inner product space $V(F)$ are orthogonal then i, $a\alpha$ is orthogonal to β and 2, 'α' is orthogonal to $a\beta$ for any $a \in F$.

Theorem:

The vectors α, β of real inner product space $V(F)$ are orthogonal iff $\|\alpha+\beta\|^2 = \|\alpha\|^2 + \|\beta\|^2$.

orthogonal set:

Let 's' be a non empty subset of an inner product space $V(F)$. The set 's' is said to be orthonormal set if.

1, $\|\alpha_i\|=1$ for each $\alpha_i \in S$ & ii, $\langle \alpha_i, \alpha_j \rangle = 0$ for $\alpha_i, \alpha_j \in S$

Gram - schmidth orthogonalisation process:

working method for finding orthonormal basis:

Let $\{\beta_1, \beta_2 \cdots \beta_n\}$ be a given basis of a finite dimensional inner product space $V(F)$. The vectors

$\alpha_1, \alpha_2 \cdots \alpha_n$ of the orthonormal basis of $V(F)$ are given by $\alpha_1 = \frac{\beta_1}{\|\beta_1\|}$

$$\alpha_2 = \frac{\gamma_2}{\|\gamma_2\|} \text{ where } \gamma_2 = \beta_2 - \langle \beta_2, \alpha_1 \rangle \alpha_1$$

$$\vdots$$

$$\alpha_n = \frac{\gamma_n}{\|\gamma_n\|} \text{ where } \gamma_n = \beta_n - \langle \beta_n, \alpha_1 \rangle \alpha_1 - \langle \beta_n, \alpha_2 \rangle \alpha_2 \cdots - \langle \beta_n, \alpha_{n-1} \rangle \alpha_{n-1}.$$

orthogonal Complement:

Let W be a non-empty subset of the inner product space $V(F)$. The orthonormal Complement of W denoted by $W^{\perp}$ is defined as the set $\{\alpha \in V \,/\, \langle \alpha, \beta \rangle = 0 \;\forall\; \beta \in W\}$.

Question Bank:

1. Find a unit vector orthogonal to $(4, 2, 3)$ in $\mathbb{R}^3$

2. In a real inner product space. if u, v are Two vectors such that $\|u\| = \|v\|$ P.T $u-v$, $u+v$ are orthogonal.

3. If α, β be two orthogonal vectors in an inner product space $V(F)$ and $\|\alpha\| = \|\beta\| = 1$ Then P.T $\|\alpha - \beta\| = d(\alpha, \beta) = \sqrt{2}$.

4. State and prove orthogonality Theorem.

5. State & prove. Bessel's Inequality.

6. State and prove. Parseval's Identity.

7. Given $\{(2,1,3), (1,2,3), (1,1,1)\}$ is a basis of $\mathbb{R}^3$, Construct an orthonormal basis.

8. Let $V(F)$ be an inner product space and $S = \{\alpha_1, \alpha_2 \ldots \alpha_m\}$ be an orthogonal subset of V consisting ~~of~~ non-zero vectors. If $\beta \in$ span $S = L(S)$ Then $\beta = \sum_{i=1}^{m} \frac{\langle \beta, \alpha_i \rangle}{\|\alpha_i\|^2} \alpha_i$.

MJ Kiran