

# V.S.M. COLLEGE, RAMACHANDRAPURAM

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TEACHING NOTES  
2017 - 2018

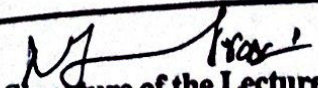
Name of the Department / Subject :

MATHEMATICS

Name of the Lecturer :

N. S. V. Kiran Kumar.

Course/Group:	III - BSc.
Paper:	V
Name of the Topic	Ring Theory.
Hours required	10
Learning Objectives	Integrate functions of several variables over curves and surfaces
Previous knowledge to be reminded	Properties of groups.
Topic Synopsis	<p>Definition of ring, Boolean ring, zero Divisors of ring, Cancellation laws</p> <p>Integral domain. Field, (Continue on the reverse side if needed)</p> <p>characteristic of a ring, Divisor (GCD) Factor, Trivial Divisors, unity element.</p>
Examples/Illustrations	Given on the Reverse side.
Additional inputs	
Teaching Aids used	Black Board.
References cited	Telugu Academy and S.Chand
Student Activity planned after the teaching	Assignments and seminar
Activity planned outside the Class room, if any	
Any other activity	

  
Signature of the Lecturer

## Definition of Ring:

Let  $R$  be a non-empty set and  $+$ ,  $\cdot$  be two binary operations in  $R$ .  $(R, +, \cdot)$  is said to be a ring if for  $a, b, c \in R$ .

$$R_1: a+b = b+a. \quad R_2: (a+b)+c = a+(b+c)$$

$$R_3: \exists 0 \in R \ni a+0 = a \text{ for } a \in R.$$

$$R_4: \exists -a \in R \ni a+(-a) = 0 \text{ for } a \in R.$$

$$R_5: (a \cdot b) \cdot c = a \cdot (b \cdot c) \text{ and } R_6: a(b+c) = ab+ac \text{ and } (b+c)a = ba+ca.$$

(oo)

Let  $R$  be a non-empty set and  $+$ ,  $\cdot$  be two binary operations in  $R$ .  $(R, +, \cdot)$  is said to be a ring if:

(i),  $(R, +)$  is a commutative group.

(ii),  $(R, \cdot)$  is semigroup. and

(iii), Distributive laws hold.

## Unity Element:

In a ring  $(R, +, \cdot)$  if  $\exists 1 \in R$  such that  $a \cdot 1 = 1 \cdot a = a$  for every  $a \in R$ . Then we say that  $R$  is a ring with unity element or Identity element.

## Definition:

In a ring  $(R, +, \cdot)$  if  $ab = ba$  for  $a, b \in R$  then we say that  $R$  is a commutative ring.

### Boolean Ring:

In a ring  $R$  if  $a^2 = a \forall a \in R$  Then  $R$  is called a Boolean ring.

### zero Divisors:

Two non-zero elements  $a, b$  of a ring  $R$  are said to be zero divisors if  $ab = 0$  where  $0 \in R$  is the zero element.

In particular ' $a$ ' is left zero divisor and ' $b$ ' is right zero divisor.

### zero Divisors:

$a \neq 0 \in R$  is a zero divisor if  $\exists b \neq 0 \in R$  s.t.  $ab = 0$

### Cancellation laws in a Ring:

In a ring  $R$ , for  $a, b, c \in R$  if  $a \neq 0$ .

$ab = ac \Rightarrow b = c$  and  $a \neq 0, ba = ca \Rightarrow b = c$  Then we say that cancellation laws hold in  $R$ .

### Integral Domain:

A commutative ring  $D$  with unity containing no zero divisors is an Integral domain.

### Field:

Let  $R$  be a commutative ring with unity element. If every non-zero element of  $R$  is invertible under multiplication Then  $R$  is a field.

### Skew field:

Let  $R$  be a ring with unity element. If every non-zero element of  $R$  is a unit Then  $R$  is a division ring.

### Characteristic of ring:

The characteristic of a ring  $R$  is defined as the least +ve integer  $p \Rightarrow pa = 0 \forall a \in R$ . In case such a positive integer  $p$  does not exist Then we say that the Ch. of  $R$  is zero (or) infinite.

## Question Bank.

1. If  $R$  is a ring and  $0, a, b \in R$  Then i,  $0a = a0 = 0$ .  
(ii),  $a(-b) = (-a)b = -(ab)$  (iii),  $(-a)(-b) = ab$  and (iv),  $a(b-c) = ab - ac$ .
2. If  $R$  is a Boolean ring Then i,  $a+a=0 \forall a \in R$ .  
(ii),  $a+b=0 \Rightarrow a=b$  (iii),  $R$  is commutative under multiplication.
3. A ring  $R$  has no zero divisors iff and only iff The Cancellation laws hold in  $R$ .
4. A field has no zero - divisors.
5. Every field is an Integral domain.
6. Every finite Integral domain is field.
7. Prove that the set  $\mathbb{Z}(i) = \{a+bi \mid a, b \in \mathbb{Z}, i^2 = -1\}$  of Gaussian Integers in an Integral domain w.r.t. addition and multiplication of numbers Is it a field?
8. Prove that  $\mathbb{Q}(\sqrt{2}) = \{a+b\sqrt{2} \mid a, b \in \mathbb{Q}\}$  is a field w.r.t ordinary addition and multiplication of numbers.
9. The characteristic of an Integral domain is either a prime or zero.
10. The set of  $2 \times 2$  matrices of the form  $\begin{bmatrix} x & y \\ -\bar{y} & \bar{x} \end{bmatrix}$  where  $x, y$  are complex numbers and  $\bar{x}, \bar{y}$  denote the complex conjugate of  $x, y$  is a skew field for compositions of matrix addition and multiplication.

# V.S.M. COLLEGE (AUTONOMOUS) RAMACHANDRAPURAM

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## TEACHING NOTES

2017 - 2018

Name of the Department / Subject :

Mathematics

Name of the Lecturer :

N. L. N. Mallika

Course/Group: II B.Sc

Paper: Paper - II

Name of the Topic: Subgroups, Cosets and Lagrange's theorem

Hours required: 08.

Learning Objectives: Analyse and demonstrate examples of subgroups, normal subgroups and quotient groups

Previous knowledge to be reminded: Properties of groups.

Topic Synopsis: Complex definition, Subgroups, Criterion for the product of two subgroups to be a subgroup, Union and Intersection of Subgroups, Definition of Coset, Properties of Cosets, Normalizer of an element of a group.  
*(Continue on the reverse side if needed)*

Examples/Illustrations: Given on the reverse side.

Additional inputs: ICT

Teaching Aids used: Black board.

References cited: S. Chand.

Student Activity planned after the teaching: Seminars, Assignment.

Activity planned outside the Class room, if any

Any other activity

N. K. N. Jadhav,  
Signature of the Lecturer

## Complex definition 1

Any subset of a group  $G$  is called a complex of  $G$ .

Eg: The set of integers is a complex of the group  $(\mathbb{R}, +)$

## Multiplication of two complexes :-

If  $M$  and  $N$  are any two complexes of a group  $G$  then  $MN = \{mn \in G \mid m \in M, n \in N\}$

clearly,  $MN \subseteq G$  and  $MN$  is called the product of the complexes  $M, N$  of  $G$ .

## Subgroups 1

Let  $(G, \cdot)$  be a group. Let  $H$  be a non-empty subset of  $G$  such that  $(H, \cdot)$  be a group. Then  $H$  is called a subgroup of  $G$ .

It is denoted by  $H \leq G$  (or)  $G \geq H$  and  $H < G$  or  $G > H$  we mean  $H \leq G$  but  $H \neq G$ .

## Union and Intersection of subgroups :-

If  $H_1$  and  $H_2$  are two subgroups of a group  $G$  then  $H_1 \cap H_2$  is also a subgroup of  $G$ .

## Cosets 1

Let  $(H, \cdot)$  be a subgroup of the group  $(G, \cdot)$   
Let  $a \in G$  then the set  $aH = \{ah \mid h \in H\}$  is called a left coset of  $H$  in  $G$  generated by 'a' and the set



## Complex definition

Any subset of a group  $G$  is called a Complex of  $G$ .

Eg: The set of integers is a Complex of the group  $(\mathbb{R}, +)$

## Multiplication of two Complexes :-

If  $M$  and  $N$  are any two Complexes of a group  $G$  then  $MN = \{mn \in G \mid m \in M, n \in N\}$

clearly,  $MN \subseteq G$  and  $MN$  is called the product of the Complexes  $M, N$  of  $G$ .

## Subgroups :-

Let  $(G, \cdot)$  be a group. Let  $H$  be a non-empty subset of  $G$  such that  $(H, \cdot)$  be a group. Then  $H$  is called a subgroup of  $G$ .

It is denoted by  $H \leq G$  (or)  $G \geq H$  and  $H < G$  or  $G > H$  we mean  $H \leq G$  but  $H \neq G$ .

## union and intersection of subgroups :-

If  $H_1$  and  $H_2$  are two subgroups of a group  $G$  then  $H_1 \cap H_2$  is also a subgroup of  $G$ .

## cosets :-

Let  $(H, \cdot)$  be a subgroup of the group  $(G, \cdot)$   
Let  $a \in G$  then the set  $aH = \{ah \mid h \in H\}$  is called a left coset of  $H$  in  $G$  generated by 'a' and the set

$Ha = \{ha/h \in H\}$  is called a right coset of  $H$  in  $G$  generated by 'a'. Also  $aH, Ha$  are called cosets of  $H$  generated by 'a' in  $G$ .

Congruence modulo  $H$  :-

Let  $(G, \cdot)$  be a group and  $(H, \cdot)$  be a subgroup of  $G$ . For  $a, b \in G$  if  $\exists a' \in H$  we say that  $a \equiv b \pmod{H}$ .

Normalizer of an element of a group :-

If 'a' is an element of a group  $G$ , then the normalizer of 'a' in  $G$  is the set of all those elements of  $G$  which commute with 'a'.

The normalizer of 'a' in  $G$  is denoted by  $N(a)$

where  $N(a) = \{x \in G / ax = xa\}$

The normalizer  $N(a)$  is a subgroup of  $G$ .

Self-conjugate element of a group :-

$(G, \cdot)$  is a group and  $a \in G$  such that  $a = x^{-1}ax, \forall x \in G$ . Then 'a' is called self conjugate element of  $G$ . A self-conjugate element is sometimes called an invariant element.

## Imp Questions

1. If  $H$  is any subgroup of a group  $G$ , then  $H^{-1} = H$ .
2.  $H$  is a non-empty complex of a group  $G$ . The necessary and sufficient condition for  $H$  to be a subgroup of  $G$  is  $a, b \in H \Rightarrow ab^{-1} \in H$  where  $b^{-1}$  is the inverse of 'b' in  $G$ .
3. The necessary and sufficient condition for a finite complex  $H$  of a group  $G$  to be a subgroup of  $G$  is  $a, b \in H \Rightarrow ab \in H$ .
4. If  $H$  and  $K$  are two subgroups of a group  $G$ , then  $HK$  is a subgroup of  $G$  iff  $HK = KH$ .
5. If  $H_1$  &  $H_2$  are two subgroups of a group  $G$  then  $H_1 \cap H_2$  is also a subgroup of  $G$ .
6. The union of two subgroups of a group is a subgroup iff one is contained in the other.
7. If  $a, b$  are any two elements of a group  $(G, \cdot)$  and  $H$  any subgroup of  $G$  then  $Ha = Hb \Leftrightarrow ab^{-1} \in H$  and  $aH = bH \Leftrightarrow a^{-1}b \in H$ .
8. Any two left (right) cosets of a subgroup are either disjoint or identical.
9. State and prove Lagrange's theorem.

*M. K. S.*

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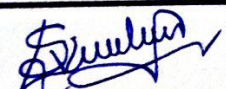
Name of the Department / Subject :

MATHEMATICS

Name of the Lecturer :

V. L. Sudhakar

Course/Group:	I B.Sc
Paper:	Paper-I
Name of the Topic	Linear Diff <sup>al</sup> Eq <sup>n</sup> with Variable Coefficients
Hours required	
Learning Objectives	Solve $f(D)y = 0$
Previous knowledge to be reminded	Find $f(D)y = 0$ solution and particular integration of $f(D)y = Q(x)$
Topic Synopsis	Method of Variation of parameters Solve Linear Diff <sup>al</sup> eq <sup>n</sup> s with non-constant coefficients by (Continue on the reverse side if needed) Method of Reduction of order method The Cauchy-Euler <sup>n</sup> eq <sup>n</sup> s & Legendre <sup>n</sup> Eq <sup>n</sup> s
Examples/Illustrations	Given on the Reverse Side
Additional inputs	ICT
Teaching Aids used	Black Board
References cited	S. Chand (Dr. M.D. Raisingana)
Student Activity planned after the teaching	Seminars, Assignments.
Activity planned outside the Classroom, if any	
Any other activity	

  
Signature of the Lecturer

## Method of Variation of Parameters:-

Second order Linear Diff<sup>al</sup> Eq<sup>n</sup> is

$$\frac{d^2y}{dx^2} + P(x) \frac{dy}{dx} + Q(x)y = R(x) \rightarrow \text{[1]}$$

$$\frac{d^2y}{dx^2} + P(x) \frac{dy}{dx} + Q(x)y = 0 \rightarrow \text{[2]}$$

The C.F =  $y_c = C_1 u(x) + C_2 v(x)$ .

Suppose  $y_p = A u(x) + B v(x)$

where  $A = -\int \frac{vR}{W}$        $B = \int \frac{uR}{W}$

where  $W = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix} = uv' - u'v$ .

The G.S of eqn [1] is  $y = C_1 u(x) + C_2 v(x) + A u(x) + B v(x)$

To solve. Linear Diff<sup>al</sup> Eq<sup>n</sup> with non-constant Coeff<sup>ts</sup> by the Method of Reduction.

## The Cauchy-Euler Eq<sup>n</sup>:-

An Eq<sup>n</sup> of the form

$$a_n x^n \frac{d^n y}{dx^n} + a_{n-1} x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_1 x \frac{dy}{dx} + a_0 y = Q(x)$$

is called a Cauchy-Euler Eq<sup>n</sup> of order n

where  $a_0, a_1, \dots, a_n$  are constants

Second order eqn is  $a_2 x^2 \frac{d^2 y}{dx^2} + a_1 x \frac{dy}{dx} + a_0 y = Q(x)$

put  $x = e^t \Rightarrow \log x = t$   
 Then  $x \frac{dy}{dx} = \frac{dy}{dt}$  or  $D \rightarrow \boxed{2}$

$$x^2 \frac{d^2y}{dx^2} = \frac{d^2y}{dt^2} - \frac{dy}{dt} \text{ (or) } x^2 D = D(D-1)$$

From  $\boxed{1}$  &  $\boxed{2}$

$$a_2 \left( \frac{d^2y}{dt^2} - \frac{dy}{dt} \right) + a_1 \left( \frac{dy}{dt} \right) + a_0 y = Q(e^t)$$

$$\Rightarrow a_2 \frac{d^2y}{dt^2} + (-a_2 + a_1) \frac{dy}{dt} + a_0 y = Q(e^t)$$

$$A_2 \frac{d^2y}{dt^2} + A_1 \frac{dy}{dt} + A_0 y = Q(e^t) \rightarrow \boxed{3}$$

where  $A_2 = a_2$ ,  $A_1 = a_1 - a_2$ ;  $A_0 = a_0$

clearly Eqn  $\boxed{3}$  is Linear Diffal Eqn with

constant coefficients. It can be solve short cut Method of  $f(D)y = Q(x)$ .

Note:  $x^3 D^3 = D(D-1)(D-2)$

The Legendre's Eqn :-

An Eqn of the form

$$(ax+b)^n \frac{d^m y}{dx^m} + (ax+b)^{n-1} \frac{d^{m-1} y}{dx^{m-1}} + \dots + (ax+b) \frac{dy}{dx} + y = Q(x)$$

Let  $ax+b = e^z \Rightarrow z = \log(ax+b)$

Then

$$(ax+b) \frac{dy}{dx} = a \cdot D$$

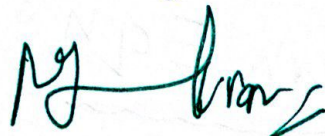
$$(ax+b)^2 \frac{dy}{dx} = a^2 \cdot D(D-1)$$

$$(ax+b)^3 \frac{dy}{dx} = a^3 D(D-1)(D-2)$$

$$(ax+b)^n \frac{d^m y}{dx^m} = a^m D(D-1)(D-2)\dots(D-(m-1))$$

## Important Questions

- ① Find the particular integral of  $y'' + 4y = 2 \tan x$  by using the method of variation of parameters
- ②  $(D^2 - 2D)y = e^x \sin x$  by the method of variation of parameters
- ③ Given  $y_1(x) = x$  as a soln of  $x^2 y'' + xy' - y = 0$ . find the general soln of  $x^2 y'' + xy' - y = x$ .
- ④ Solve  $x^2 y'' + 3xy' + y = 0$  a.T  $y_1 = \frac{1}{x}$  is a soln
- ⑤ Solve  $x^2 y''' = 2y'$
- ⑥ solve  $x^2 y'' + 7xy' - 7y = x^2 + x$
- ⑦  $x^2 y'' - xy' + y = 2 \log x$
- ⑧  $x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = 11x^3$ .
- ⑨  $[(3x+2)^2 D^2 + 3(3x+2)D - 36]y = 3x^2 + 4x + 1$ ,
- ⑩  $(1+x^2) \left( \frac{d^2 y}{dx^2} \right) + (1+x) \left( \frac{dy}{dx} \right) + y = 4 \cos \log(1+x)$ .

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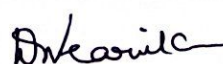


## TEACHING NOTES 2017 - 2018

Name of the Department / Subject : **MATHEMATICS**

Name of the Lecturer : **D. S. V. Hasika**

Course/Group:	I B.sc
Paper:	Paper - I
Name of the Topic	The sphere
Hours required	19
Learning Objectives	Find the Equation of spheres and properties of spheres
Previous knowledge to be reminded	properties of the straight line
Topic Synopsis	<p>Definition of sphere, area of the sphere, Intersection of a plane and sphere, great circle, Tangent plane, polar plane, plane of contact, power point, (Continue on the reverse side if needed) conjugate points, Intersection of two spheres, orthogonal spheres, orthogonal spheres, Radical plane, co-axial system</p>
Examples/Illustrations	Given on the reverse side
Additional inputs	ICT
Teaching Aids used	Black Board
References cited	S. Chand
Student Activity planned after the teaching	Assignment, Seminar
Activity planned outside the Classroom, if any	
Any other activity	

  
**Signature of the Lecturer**

Sphere: The set of points in a space which are at a constant distance  $a (a \geq 0)$  from a fixed point  $c$  is called a sphere

→ Equation to the sphere with center  $(x_1, y_1, z_1)$  and radius  $a$  is  $(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2 = a^2$

→ Equation to sphere is of the form  $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$  where radius =  $\sqrt{u^2 + v^2 + w^2 - d}$  and center  $c (-u, -v, -w)$ .

concentric sphere:

sphere with the same center are known as concentric sphere.

→ A plane section of a sphere is a circle

→ great circle:

If a plane  $\pi$  passes through the center of a sphere  $S$ , then the plane section of the sphere is called a great circle.

→ If two spheres intersect, then the locus of the set of points of intersection is a circle

→ length of the tangent line from  $(x_1, y_1, z_1)$  to the sphere  $S=0$  is  $\sqrt{S_{11}}$

→ Equation of the plane of contact of all point  $(x_1, y_1, z_1)$  with respect to the sphere  $S=0$  of non-zero radius  $a$  is  $S_1 = 0$ .

→ The Equation of the polar plane of the point  $(x_1, y_1, z_1)$  with respect to the sphere  $S=0$  is  $S_1 = 0$ .

→ If  $x^2 + y^2 + z^2 - a^2 = 0$  is a sphere, then the pole of the plane  $lx + my + nz = p$  is  $\left( \frac{a^2 l}{p}, \frac{a^2 m}{p}, \frac{a^2 n}{p} \right)$

## Angle of intersection of sphere:

$P$  is common point to 2 spheres  $S_1, S_2$

Any angle ' $\theta$ ' between the tangent planes at  $P$  to two sphere is called an angle of intersection of the sphere.  $S_1, S_2$  at  $P$  then other angle between the sphere is  $\pi - \theta$ .

→ Equation to the radical plane of sphere

$$S = 0, S' = 0 \text{ is } S - S' = 0.$$

→ If  $S = 0$  is a sphere and  $U = 0$  is a plane, then the Equation  $S + AU = 0$  represent a co-axial system of sphere with radical plane  $U = 0$ .

*NJ Bora*

## Important questions

- ① The plane of  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$  meets the axes in A, B, C. Find the Equation of the circumcircle of  $\Delta ABC$  and hence Find its center
- ② Find great circle (or) small circle  
 $x^2 + y^2 + z^2 - 4x + 6y - 8z + 4 = 0, \quad x + y + z = 3$
- ③ Show that the plane  $2x - 2y + z + 12 = 0$  touches the sphere  $x^2 + y^2 + z^2 - 2x + 4y + 2z - 3 = 0$  and Find the point of contact.
- ④ Find the pole of the plane  $10x - 2y - 5z - 2 = 0$  with respect to sphere  $x^2 + y^2 + z^2 - 6x + 2y - 2z + 1 = 0$ .
- ⑤  $S_1, S_2$  are two intersecting spheres;  $r_1, r_2$  are their respective radii and  $d$  is the distance between their centers. If  $P$  is a common point to  $S_1, S_2$  then an angle  $\theta$  of intersection of the sphere  $S_1, S_2$  at  $P$  is given by
- $$\cos \theta = \pm \frac{(r_1^2 + r_2^2 - d^2)}{2r_1 r_2}$$
- ⑥ If  $r_1, r_2$  are the radii of two orthogonal spheres, then the radius of the circle of their intersection is  $\frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$

M. K. S.

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TEACHING NOTES  
2017 - 2018

Name of the Department / Subject : MATHEMATICS

Name of the Lecturer : G. N. Uma Maheswari

Course/Group: III B.Sc

Paper: 2P

Name of the Topic

orthogonality.

Hours required

8.

Learning Objectives

Previous knowledge to be reminded

Properties of inner product space.

Topic Synopsis

orthogonal  
orthogonal set  
orthonormal.  
Gram Schmidt orthogonalisation process.  
(Continue on the reverse side if needed)  
orthogonal complement and Important  
Theorems.

Examples/Illustrations

Given on the reverse side.

Additional inputs

Teaching Aids used

Black Board.

References cited

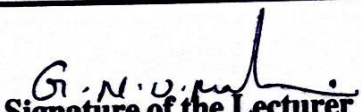
Telugu Academy & S.Chand.

Student Activity planned after the teaching

Assignments & Seminars

Activity planned outside the Classroom, if any

Any other activity

  
Signature of the Lecturer

## Orthogonal:

Let  $V(F)$  be an inner product space and  $\alpha, \beta \in V$ .  
 $\alpha$  is said to be orthogonal to  $\beta$  if  $\langle \alpha, \beta \rangle = 0$ .  
 $\alpha$  is orthogonal to  $\beta \Rightarrow \langle \alpha, \beta \rangle = 0$   
 $\langle \alpha, \beta \rangle = 0 \Leftrightarrow \beta$  is orthogonal to  $\alpha$ .

## Theorem:

If the vectors  $\alpha, \beta$  of an inner product space  $V(F)$  are orthogonal then  $i, \alpha$  is orthogonal to  $\beta$  and  $\alpha, \beta$  is orthogonal to  $a\beta$  for any  $a \in F$ .

## Theorem:

The vectors  $\alpha, \beta$  of real inner product space  $V(F)$  are orthogonal iff  $\|\alpha + \beta\|^2 = \|\alpha\|^2 + \|\beta\|^2$ .

## Orthogonal Set:

Let  $S$  be a nonempty subset of an inner product space  $V(F)$ . The set  $S$  is said to be orthonormal set if.

1.  $\|\alpha_i\| = 1$  for each  $\alpha_i \in S$  &  $i, j, \langle \alpha_i, \alpha_j \rangle = 0$  for  $\alpha_i, \alpha_j \in S$

Gram-Schmidt orthogonalisation process:

working method for finding orthonormal basis:

Let  $\{\beta_1, \beta_2, \dots, \beta_n\}$  be a given basis of a finite dimensional inner product space  $V(F)$ . The vectors



$\alpha_1, \alpha_2, \dots, \alpha_n$  of the orthonormal basis of  $V(F)$  are given by  $\alpha_1 = \frac{\beta_1}{\|\beta_1\|}$

$$\alpha_2 = \frac{\beta_2}{\|\beta_2\|} \text{ where } \beta_2 = \beta_2 - \langle \beta_2, \alpha_1 \rangle \alpha_1,$$

$$\alpha_n = \frac{\beta_n}{\|\beta_n\|} \text{ where } \beta_n = \beta_n - \langle \beta_n, \alpha_1 \rangle \alpha_1 - \langle \beta_n, \alpha_2 \rangle \alpha_2 - \dots - \langle \beta_n, \alpha_{n-1} \rangle \alpha_{n-1}.$$

Orthogonal Complement:

Let  $\omega$  be a non-empty subset of the inner product space  $V(F)$ . The orthogonal complement of  $\omega$  denoted by  $\omega^\perp$  is defined as the set  $\{\alpha \in V \mid \langle \alpha, \beta \rangle = 0 \ \forall \beta \in \omega\}$ .

## Question Bank:

1. Find a unit vector orthogonal to  $(4, 2, 3)$  in  $\mathbb{R}^3$
2. In a real inner product space. if  $u, v$  are two vectors such that  $\|u\| = \|v\|$  p.t  $u-v, u+v$  are orthogonal.
3. If  $\alpha, \beta$  be two orthogonal vectors in an inner product space  $V(F)$  and  $\|\alpha\| = \|\beta\| = 1$  Then p.t  $\| \alpha - \beta \| = d(\alpha, \beta) = \sqrt{2}$ .
4. State and prove orthogonality Theorem.
5. State & prove. Bessel's Inequality.
6. State and prove. Parseval's Identity.
7. Given  $\{(2, 1, 3), (1, 2, 3), (1, 1, 1)\}$  is a basis of  $\mathbb{R}^3$ , Construct an orthonormal basis.
8. Let  $V(F)$  be an inner product space and  $S = \{\alpha_1, \alpha_2, \dots, \alpha_m\}$  be an orthogonal subset of  $V$  consisting of non-zero vectors. If  $\beta \in \text{span } S = \langle S \rangle$  Then 
$$\beta = \sum_{i=1}^m \frac{\langle \beta, \alpha_i \rangle}{\|\alpha_i\|^2} \alpha_i$$

MJ