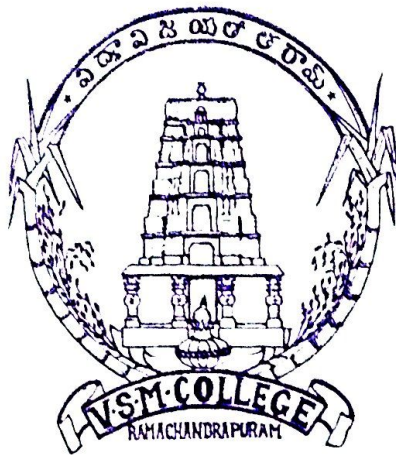


V.S.M. COLLEGE (AUTONOMOUS) RAMACHANDRAPURAM

(Re - Accredited by NAAC with 'B' Grade)



TEACHING NOTES
2016 - 2017

Name of the Department / Subject : *Mathematics*

Name of the Lecturer : *N.S.V. Kiran Kumar*

Course/Group: III B.S.C - A, B

Paper: IV

Name of the Topic

Error in numerical computations

Hours required

04

Learning Objectives

use the definition of convergence as they apply to sequences, series and functions

Previous knowledge to be reminded

using scientific calculator

Topic Synopsis

Absolute error
Relative error
Percentage error
(Continue on the reverse side if needed)
Error in a series expansion
General error formula
Rounding - off

Examples/Illustrations

Given on the reverse side

Additional inputs

Teaching Aids used

Black Board

References cited

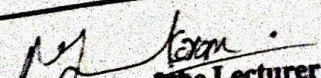
Error in numerical computations telugu academy & s. chand.

Student Activity planned after the teaching

Assignments & seminars

Activity planned outside the Class room, if any

Any other activity


Signature of the Lecturer

Absolute Error :-

Absolute error is the numerical difference between the true value of a quantity and its approximate value. If x is the value of a quantity and x' its approximate value, then $|x - x'|$ is called the absolute error. E_A

$$E_A = |x - x'|$$

Relative Error :-

The relative error E_R is defined by

$$E_R = \frac{E_A}{x} = \left| \frac{x - x'}{x} \right|$$

Percentage error :-

The percentage error E_P is defined by

$$E_P = E_R \times 100 = \left| \frac{x - x'}{x} \right| \times 100$$

General error formula :-

$$E_R = \frac{\Delta u}{u} = \frac{\delta f}{\delta x_1} \cdot \frac{\Delta x_1}{u} + \frac{\delta f}{\delta x_2} \cdot \frac{\Delta x_2}{u} + \dots + \frac{\delta f}{\delta x_n} \cdot \frac{\Delta x_n}{u}$$

Error in a series expansion :-

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!} h + \frac{f''(x_0)}{2!} h^2 + \dots + \frac{f^{(n)}(x_0)}{n!} h^n + R_{n+1}(x)$$

$$\text{where } R_{n+1}(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!} h^{n+1} \cdot o(h^{n+1})$$

By $o(h^{n+1})$, we mean that $\lim_{h \rightarrow 0} \frac{R_{n+1}(x)}{h^{n+1}} = \text{constant}$

The zeroth order of approximation of $f(x)$ becomes

$$f(x) = f(x_0) + o(h)$$

where $o(h) = \frac{f'(x_0)}{1!} h$ for x near x_0

The first order of approximation of $f(x)$ becomes

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!} h + o(h^2)$$

where $o(h^2) = \frac{f''(x_0)}{2!} h^2$

Rounding - off :-

Frequently we come across numbers with a large number of digits E.g. $13/7 = 1.8571429$

In practice, it will be necessary to cut them to a usable number of digits such as 1.8 or 1.86. This process of cutting off unwanted digits is called rounding - off

Working Rule :-

To rounded-off a number to n

Significant digits.

1. Discard all digits to the right of the n th digit

2. If the discarded number is

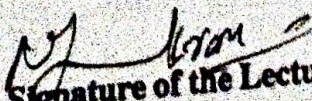
↳ less than half a unit in the n th place, leave the n th digit unchanged.

↳ Greater than half a unit in the n th place, increases the n th unit by unity.

Question Bank :-

1. Find the E_A if $\frac{2}{3}$ is approximated to 0.667
2. Find the E_R if $\frac{2}{3}$ is approximated to 0.667
3. Find the percentage error if 625.483 is approximated to 3 significant figures
4. If $R = \frac{4x^2y^3}{z^4}$ and errors in x, y, z be 0.001, s.t the maximum relative error of $x=y=z=1$ is 0.009
5. If $x = 0.64$ and is correct to 2 decimal places, then find the relative error and then % error
6. An approximate value of π is given by $\lambda_1 = \frac{22}{7} \approx 3.14285714$ and its ~~abs~~ value $\lambda = 3.14159265$ is true. Find the absolute error and the relative error.
7. Using the Maclaurin series expansion for e^x approximate e^1 correct upto 5 decimal places

Course/Group:	III - BSc - A, B
Paper:	54
Name of the Topic	Vector Spaces
Hours required	11
Learning Objectives	Vector space, Sum of Subspaces, Linear Span, L.I, L.D of vectors, Linear Combination etc
Previous knowledge to be reminded	Properties of groups
Topic Synopsis	<p>Vector space; zero vector space; Vector subspaces Linear sum of two subspaces</p> <p><i>(Continue on the reverse side if needed)</i> linear combination of vectors linear span linear dependence of vectors linear independence of vector</p>
Examples/Illustrations	Given on the Reverse side
Additional inputs	
Teaching Aids used	Black Board
References cited	Telugu Academy & S.Chand
Student Activity planned after the teaching	Assignments & seminar
Activity planned outside the Class room, if any	
Any other activity	


Signature of the Lecturer

Vector spaces:-

Let V be a non-empty set whose elements are called vectors. Let F be any set whose elements are called scalars. Where $(F, +, \dots)$ is a field

The set V is said to be a vector space if

i) $(V, +)$ is an abelian group

ii) V is closed under scalar multiplication

iii) a) $a(\alpha + \beta) = a\alpha + a\beta$ b) $(a+b)\alpha = a\alpha + b\alpha$ c) $(ab)\alpha = a(b\alpha)$

d) $1 \cdot \alpha = \alpha$

Null space or zero vector space:-

The vector space having only one zero vector " 0 " is called the zero vector space or null space

Vector subspaces:- Let $V(F)$ be a vector space and $W \subseteq V$. Then W is said to be a subspace of V if W itself is a vector space over F with the same operations of vector addition & scalar multiplication in V

Linear sum of two subspaces:- Let W_1 and W_2 be two subspaces of the vector space $V(F)$. Then the linear sum of the subspaces W_1 and W_2 denoted by $W_1 + W_2$ is the set of all sum $\alpha_1 + \alpha_2$ s.t. $\alpha_1 \in W_1, \alpha_2 \in W_2$ i.e.,
 $W_1 + W_2 = \{ \alpha_1 + \alpha_2 \mid \alpha_1 \in W_1, \alpha_2 \in W_2 \}$

Linear combination of vector:- Let $V(F)$ be a vector space, if $\alpha_1, \alpha_2, \dots, \alpha_n \in V$ then any vector $\gamma = a_1\alpha_1 + a_2\alpha_2 + \dots + a_n\alpha_n$ where $a_1, a_2, a_3, \dots, a_n \in F$ is called linear combination of vectors

Linear span of a set:-

Let S be a non-empty subset of a vector space $V(F)$. The linear span of S is the set of all possible

linear combinations of all possible finite subset of S

The linear span of S is denoted by $L(S)$

1) $S \subseteq L(S)$

2) S may be a finite set but $L(S)$ is infinite set

3) If S is an empty subset of V then we define $L(S) = \{0\}$

Linear dependence of vectors:-

Let $V(F)$ be a vector space. A finite subset $\{x_1, x_2, \dots, x_n\}$

of vectors of V is said to be a linear dependence of

vectors (L-D) set if there exists scalars $a_1, a_2, \dots, a_n \in F$

not all zero, such that $a_1 x_1 + a_2 x_2 + \dots + a_n x_n = \bar{0}$

Linear independence of vectors:-

Let $V(F)$ be a vector space. A finite subset $\{x_1, x_2, \dots, x_n\}$

of vectors of V is said to be a linear independent

(L-I) set if there exists scalars $a_1, a_2, \dots, a_n \in F$

$a_1 = 0; a_2 = 0; \dots, a_n = 0$ such that $a_1 x_1 + a_2 x_2 + \dots + a_n x_n = \bar{0}$

Question Bank:-

① Let $V(F)$ be a vector space

↓ If $a, b \in F$ & $\alpha \in V$ where $\alpha \neq \bar{0}$ then $a\alpha = b\alpha \Rightarrow a = b$

∴ If $a \in F$ where $a \neq 0$ & $\alpha, \beta \in V$ then $a\alpha = a\beta \Rightarrow \alpha = \beta$.

② Let $V(F)$ be a v.s and $w \subseteq V$. The necessary and sufficient conditions for w to be a subspace of V are

↳ $\alpha \in w; \beta \in w \Rightarrow \alpha - \beta \in w$ ∴ $a \in F; \alpha \in w \Rightarrow a\alpha \in w$

③ Let $V(F)$ be a v.s. A non empty set $w \subseteq V$.

The necessary and sufficient condition for w to

be a subspace of V is (i) $a, b \in F : \alpha, \beta \in V$
 $\Rightarrow \alpha\alpha + \beta\beta \in W$

④ A non-empty set W is a subset of v.s $V(F)$
 W is subspace of V if and only if $a \in F$ &
 $\alpha, \beta \in V \Rightarrow \alpha\alpha + \beta\beta \in W$

⑤ The linear span $L(S)$ of any subset S of a vector space $V(F)$ is a subspace of $V(F)$.

⑥ If S is a subset of a v.s $V(F)$ then p.t
 \downarrow S is subspace of $V \Leftrightarrow L(S) = S$
 $\therefore L(L(S)) = L(S)$

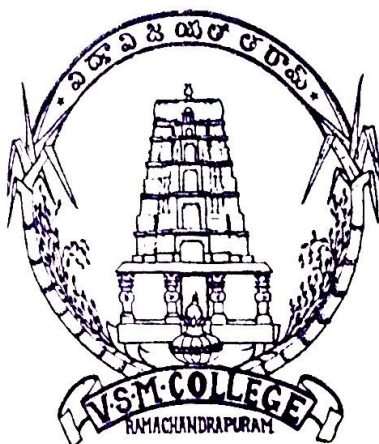
⑦ If S, T are the subsets of a v.s $V(F)$ then

$$V \quad S \subseteq T \Rightarrow L(S) \subseteq L(T)$$

$$\therefore L(S \cup T) = L(S) + L(T)$$

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TEACHING NOTES
2016 - 2017

Name of the Department / Subject :

Mathematics

Name of the Lecturer :

G.N.U. Maheswari

Course/Group:	IBSC
Paper:	Paper - I.
Name of the Topic	Differential Equations of first order & first degree
Hours required	08.
Learning Objectives	Solve the first order, first degree D.E.
Previous knowledge to be reminded	Formation of Differential Equations, Variables separable, Homogeneous Differential Equations.
Topic Synopsis	Linear differential Equations, Bernoulli's differential Equations, Exact Differential Equations, Integrating Factors, (Continue on the reverse side if needed) Reduce to exact Differential Equations (Methods), change of variables.
Examples/Illustrations	Given on the reverse side.
Additional inputs	ICT, PPT.
Teaching Aids used	Black Board.
References cited	S. Chand.
Student Activity planned after the teaching	Seminars, Assignment.
Activity planned outside the Class room, if any	
Any other activity	

Dr. N. O. Mahi
Signature of the Lecturer

Linear Differential Equations:

An Equation of the form $\frac{dy}{dx} + py = Q$. is called linear differential Equation.

where p, Q are functions in 'x'

General solution of linear differential Equation is

$$y(I.F) = \int Q(I.F) dx + C. \text{ where } I.F = e^{\int P dx}.$$

Bernoulli's Differential Equation:

An Equation of the form $\frac{dy}{dx} + py = Qy^n \rightarrow ①$

is called Bernoulli's differential Equations.

where $n \neq 0$ and $n \neq 1$.

solution:

$$y^{-n} \frac{dy}{dx} + py^{1-n} = Q \rightarrow ②$$

$$\text{let } y^{1-n} = v \Rightarrow (1-n)y^{-n} \frac{dy}{dx} = \frac{dv}{dx} \\ \Rightarrow y^{-n} \frac{dy}{dx} = \frac{1}{1-n} \frac{dv}{dx} \} \rightarrow ③$$

from ② & ③. $\frac{dv}{dx} + (1-n)p v = (1-n)Q$ This is linear differential Equation.

Exact Differential Equation:

An Equation $Mdx + Ndy = 0$ is called Exact if $f(x,y) \ni Mdx + Ndy = d[f(x,y)]$.

Note:

$Mdx + Ndy = 0$ is exact iff $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

Solution of Exact D.E:

$$\int M dx + \int (\text{terms of } N \text{ not containing } x) dy = C.$$

Integrating Factor: Method-I

Let $M dx + N dy = 0$ be not an exact differential Equation. If $M dx + N dy = 0$ can be made exact by multiplying it with a suitable function $\mu(x, y) \neq 0$. Then $\mu(x, y)$ is called an Integrating Factor of $M dx + N dy = 0$.

Method-II:

$M dx + N dy = 0$ is not exact and homogeneous. Then

$$I.F = \frac{1}{Mx + Ny} \quad \text{where } Mx + Ny \neq 0.$$

Method-III:

$M dx + N dy = 0$ is not exact and $M = y f(x, y)$ and $N = x g(x, y)$

$$\text{Then } I.F = \frac{1}{Mx - Ny} \quad \text{where } Mx - Ny \neq 0.$$

Method-IV:

$M dx + N dy = 0$ is not exact and $\frac{1}{N} \left[\frac{dM}{dy} - \frac{dN}{dx} \right] = f(x)$.

$$\text{(or) } k. \text{ Then } I.F = e^{\int f(x) dx} \quad \text{(or) } e^{\int k dx}.$$

Method-V:

$M dx + N dy = 0$ is not exact and $\frac{1}{M} \left[\frac{dN}{dx} - \frac{dM}{dy} \right] = g(y)$ (or) k .

$$\text{Then } I.F = e^{\int g(y) dy} \quad \text{(or) } e^{\int k dy}.$$

Method-VI:

$M dx + N dy = 0$ is not exact and $M dx + N dy = 0$ is of


$$\text{the form } x^h y^k (m y dx + n x dy) + x^c y^d (p y dx + q x dy) = 0.$$

$$\text{Then } I.F \text{ is } x^h y^k.$$

Important Questions.

- 1) solve. $x \frac{dy}{dx} + 2y - x^v \log x = 0.$
- 2) obtain the equation of the curve satisfying the D.E
 $(1+x^v) \frac{dy}{dx} + 2xy - 4x^v = 0$ and passing through the origin.
- 3) solve. $(1+y^v) dx = (\tan(y-x)) dy.$
- 4) solve $x \frac{dy}{dx} + y = y^v \log x.$
- 5) solve. $\frac{dy}{dx} (x^v y^3 + 2y) = 1.$
- 6) solve $(1+e^{xy}) dx + e^{xy} (1 - x/y) dy = 0$
- 7) solve. $x^2 y dx - (x^3 + y^3) dy = 0.$
- 8) solve $y(xy + 2x^v y^v) dx + x(xy - x^v y^v) dy = 0$
- 9) solve $(x^v y^v + xy + 1) y dx + (x^v y^v - xy + 1) x dy = 0$
- 10) solve $[y + \frac{y^3}{3} + \frac{x^v}{2}] dx + \frac{1}{4} (x + xy^v) dy = 0.$
- 11) solve. $(xy^v - x^v) dx + (3x^v y^v + x^v y - 2x^3 + y^v) dy = 0.$

Course/Group:	IB.SC
Paper:	Paper - I
Name of the Topic	The plane.
Hours required	18.
Learning Objectives	Find the Equation of the plane.
Previous knowledge to be reminded	The coordinates.
Topic Synopsis	Def. of the planes, Normal form, Intercept form, Equation of the plane, distance between two parallel planes, perpendicular distance from a pt to plane, angle b/w two planes, Bisecting plane of any two planes.
Examples/Illustrations	Given on the Reverse side.
Additional inputs	P.C.T.
Teaching Aids used	Black Board.
References cited	Schand.
Student Activity planned after the teaching	Seminar, Assignment.
Activity planned outside the Class room, if any	
Any other activity	

G.N.C. 
Signature of the Lecturer

Plane:

A plane is a surface such that if any two points are taken on it. The line joining them lies wholly on the surface.

An equation of the form $ax+by+cz+d=0$, $a, b, c \in \mathbb{R}$, $a^2+b^2+c^2 \neq 0$ is called a real first degree equation in x, y, z where a, b, c are d.r.'s of the normal of the plane.

→ Normal form of the plane is $lx+my+nz=p$.

→ Intercept form of the plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

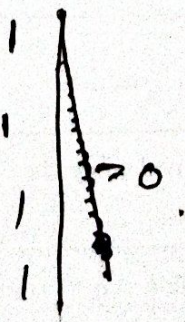
→ The equation of the plane passing through the origin is of the form $ax+by+cz=0$.

→ The equation of any plane passing through the point (x_1, y_1, z_1) is of the form $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$.

→ If $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ are non-collinear then the equation of the plane determined

by A, B, C is.

$$\begin{vmatrix} x & y & z \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$



→ perpendicular distance from (x_1, y_1, z_1) to the plane

$ax+by+cz+d=0$ is $\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$.

→ The Equation of the plane passing through the point (x_1, y_1, z_1) and parallel to the plane $ax+by+cz+d=0$ is,
 $a(x-x_1) + b(y-y_1) + c(z-z_1) = 0.$

→ The foot of the \perp from $L(p, q, r)$ from a given point $P(x_1, y_1, z_1)$ on to the plane $ax+by+cz+d=0$ is given by

$$\frac{p-x_1}{a} = \frac{q-y_1}{b} = \frac{r-z_1}{c} = \frac{-(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}.$$

→ If $L'(p', q', r')$ is the image of $P(x_1, y_1, z_1)$ w.r.to the plane $ax+by+cz+d=0$ Then,

$$\frac{p'-x_1}{a} = \frac{q'-y_1}{b} = \frac{r'-z_1}{c} = \frac{-2(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}.$$

→ Angle between two planes $a_1x+b_1y+c_1z+d_1=0$ & $a_2x+b_2y+c_2z+d_2=0$ is θ Then,

$$\cos \theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2} \sqrt{a_2^2+b_2^2+c_2^2}}$$

→ The Equation of the planes Bisecting the angle between the planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0.$

$$\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}} = \pm \frac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}}.$$

Important question

- 1) Find the Equation of the plane passing through the points $(-1, 2, -2)$ $(0, 1, 1)$ $(1, 1, 2)$ does this plane pass through $(-1, 1, 0)$
- 2) If the sum of the reciprocals of the intercepts made by a variable plane on the coordinate axes is a non-zero constant. Then s.t it passes through a fixed point in all its position.
- 3) Find the Equation of the two planes which passes through the points $(0, 4, -3)$ & $(6, -4, 3)$ and which cut off from the axes intercepts whose sum is zero.
- 4) A variable plane is at a constant distance p from the origin and meets the coordinate axes in A, B, C . s.t the locus of the Centroid of the tetrahedron $OABC$ is $x^2 + y^2 + z^2 = 16p^2$.
- 5) s.t the plane $14x - 8y + 13z = 0$ bisects the obtuse angle between the planes $3x + 4y - 5z + 1 = 0$ & $5x + 12y - 13z = 0$.

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TEACHING NOTES
2016 - 2017

Name of the Department / Subject : *Mathematics*

Name of the Lecturer : *J. V. Sathibabu*

Course/Group:	IBSC
Paper:	Paper - I
Name of the Topic	Orthogonal Trajectories & Polar Coordinates
Hours required	04
Learning Objectives	Solve Orthogonal Trajectories & Polar Coordinates
Previous knowledge to be reminded	Properties of Differentials & Integrals
Topic Synopsis	orthogonal Trajectories, self orthogonal Trajectories, Polar Coordinates (Continue on the reverse side if needed)
Examples/Illustrations	Given on the reverse side
Additional inputs	TeT
Teaching Aids used	Black Board
References cited	S. Choud
Student Activity planned after the teaching	Seminars, Assignment
Activity planned outside the Class room, if any	
Any other activity	

K. V. Dey
Signature of the Lecturer

Trajectory:-

If a curve 'c' cuts every member of a given family of curves 'T' according to some specified law, then the curve 'c' is called trajectory of the given family of curves 'T'.

Orthogonal Trajectory:- If a curve 'c' cuts every member of a given family of curves 'T' at a right angle, then the curve 'c' is called an orthogonal trajectory of the family 'T'.

Self Orthogonal Family of Curves:-

If each member of a given family of curves ~~is~~ cuts every other member of the family at right angle, then the given family of curves is said to be self orthogonal.

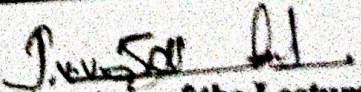
Polar Coordinates:-

If $f(r, \theta, c) = 0$, c being the parameter, is the polar Eqⁿ of the family of curves, then the diff. Eqⁿ of the family of the orthogonal Trajectories is $f(r, \theta - 90^\circ, \frac{dr}{d\theta}) = 0$

IMP Questions

- ① Find the orthogonal Trajectories of the family of curves $y = \frac{1}{\log c_1 x}$ where c_1 is parameter.
- ② Find the orthogonal Trajectories of the family of curves $x^{2/3} + y^{2/3} = a^{2/3}$, where 'a' is parameter
- ③ Find the orthogonal trajectories of the family of coaxial of circles $x^2 + y^2 + 2gx + c = 0$, where 'g' is parameter
- ④ S.T The family of confocal conics $\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} = 1$ is self orthogonal, where λ is the parameter.
- ⑤ Find the orthogonal Trajectories of the family of curves $r = a(1 - \cos \theta)$ where 'a' is the parameter
- ⑥ Find the orthogonal Trajectories of the family of curves $r = \frac{2a}{1 + \cos \theta}$ where 'a' is parameter

Course/Group:	I BSc
Paper:	Paper - I.
Name of the Topic	Differential Equation of first order and H. d.e.
Hours required	12.
Learning Objectives	Solve the first order, but not first degree differential equation.
Previous knowledge to be reminded	Find the roots.
Topic Synopsis	Form of D.E of first degree and lower higher degree and solvable for p, x, y and problems Clairaut's eqn. (Continue on the reverse side if needed)
Examples/Illustrations	Given on the reverse side
Additional inputs	ICT
Teaching Aids used	Black board
References cited	S. Chand.
Student Activity planned after the teaching	Seminars, Assignment.
Activity planned outside the Class room, if any	
Any other activity	


Signature of the Lecturer

An equation of the form: $\left(\frac{dy}{dx}\right)^n + P_1 \left(\frac{dy}{dx}\right)^{n-1} + \dots + P_{n-1} \left(\frac{dy}{dx}\right) + P_n = 0$

is called first order, n th degree differential Eqⁿ where P_1, P_2, \dots, P_n are functions of x and y

write $p = \frac{dy}{dx}$ in Eqⁿ (1) can be written as

$$p^n + P_1 p^{n-1} + P_2 p^{n-2} + \dots + P_{n-1} p + P_n = 0. \text{ Eq (1) can also be written as } f(x, y, p) = 0.$$

Method of finding a general sol. of $F(x, y, p) = 0$.

method - (i) solvable for p in terms of x and y

Let $F(x, y, p) = 0$ be the n th degree polynomial then

$$F(x, y, p) = [p - f_1(x, y)][p - f_2(x, y)] \dots [p - f_n(x, y)]$$

If $\phi: (x, y, c_i) = 0$ is a sol. of $p - f_i(x, y) = 0$ for $i = 1, 2, \dots, n$

then $\phi(x, y, c_1), \phi(x, y, c_2), \dots, \phi(x, y, c_n) = 0$ are the solutions of $F(x, y, p)$

method: - (ii) Solvable for "y"

Suppose $f(x, y, p) = 0$ is the given D.E and it is possible to find y in terms of "x" and $p = y = f(x, p)$ \rightarrow (1)

$$= \frac{dy}{dx} + \frac{\partial f}{\partial p} - \frac{dp}{dx} \rightarrow (2)$$

where $p = \frac{dy}{dx}$

which is a first order D.E in p and x and its solution of the form $\phi(x, p, c) = 0 \rightarrow$ (2)

Eliminating p from Eq ① and ② we get a solution x, y and c which is the required general solution.

Method - III. Solvable for "x"

If "x" is not of first degree in the given D.E $f(x, y, p) = 0$ then it is Solvable for "x" and it can be written as a function of y and p .
 $\therefore x = f(x, y) \rightarrow 0$ diff w.r.t "y" we have,

$$\frac{dx}{dy} = \frac{\partial f}{\partial y} + \frac{\partial f}{\partial p} \cdot \frac{\partial p}{dy} \Rightarrow \frac{1}{p} = \frac{\partial f}{\partial y} + \frac{\partial f}{\partial p} \cdot \frac{\partial p}{\partial y}$$

which is a first order differential equation in p and y and its sol. is of the form $\phi(y, p, c) = 0$

Eliminating p from ① and ② we get a relation between x, y and c which is the required G.S

Clairaut's Equations :-

$y = xp + f(p)$ is called Clairaut's

Equation. A solution of Clairaut's eqn is

$$y = cx + f(c)$$

Important Questions

1. Solve $p^2 + 2py \cot x = y^2$.

2. Solve $x^2 \left(\frac{dy}{dx}\right)^2 - 2xy \frac{dy}{dx} + 2y^2 - x^2 = 0$.

3. Solve $x^2 \left(\frac{dy}{dx}\right)^2 + xy \frac{dy}{dx} - by^2 = 0$.

4. Solve $y^2 \log y = xpy + p^2$

5. Solve $2px = 2\tan y + p^3 \cos^2 y$

6. Solve $y + px = p^2 x^4$

7. Solve $y = 2px + x^2 p^4$

8. Solve $y = x^2 p^2 + p$.

9. Solve $(y - xp)(p - 1) = p$.

10. $(py + x)(px - y) = 2p$.